THE UNIVERSITY OF ALABAMA

THIRD SEMI-ANNUAL REPORT ON NsG-381

IN SUPPORT OF RESEARCH

IN THE AEROSPACE PHYSICAL SCIENCES

March 1, 1964 - August 31, 1964

UNIVERSITY, ALABAMA

October 27, 1964

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Abstract

This is the third semi-annual report relating to grant NsG-381 which was made pursuant to a proposal of the President of the University of Alabama dated August 31, 1962 (revised December 12, 1962). NsG-381, a grant of $600,000 to the University of Alabama to be apportioned $300,000 for the first year, $200,000 for the second year, and $100,000 for the third year, was authorized by the National Aeronautics and Space Administration on March 1, 1963. Effective March 1, 1964, NASA authorized a supplement to this grant in the amount of $300,000 to the Research Institute, with $100,000 apportioned for the year beginning March 1, 1964. Thus an amount of $300,000 is available for the second grant year, plus an addition of $54,941 carried forward from the first grant year, making a total of $354,941 available.

This report is a resume of the accomplishments during the period March 1, 1964 to August 31, 1964, under this general purpose research grant. The report contains a review of the progress and growth of capability of the University of Alabama Research Institute in terms of research accomplished and in progress, facilities, manpower and contract performance, information on related educational and community industrial development activities, a brief outlook, and a report of the Comptroller of the University of Alabama accounting for the expenditure of funds for the reporting period.

Progress toward strengthening the educational value and research capabilities of the University of Alabama activities in Huntsville which was reported in the first two reports has accelerated in the past six months, and the strengthening of the mutually supporting relationships between the main campus activities and those in Huntsville has continued steadily.

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THIRD SEMI-ANNUAL REPORT ON NsG-381
IN SUPPORT OF RESEARCH IN THE AEROSPACE PHYSICAL SCIENCES
March 1, 1964 - August 31, 1964

1.0 Introduction

This report outlines the effort during the period March 1, 1964 to August 31, 1964 supported wholly or in part by NsG-381. It reports efforts of the University of Alabama and its Research Institute (U.A.R.I.) to develop capabilities for providing support to the National Aeronautics and Space Administration by conducting experimental and theoretical research, by furnishing on-the-project training of graduate students in research and by facilitating the development of instructional programs in science and engineering at the University of Alabama, particularly the graduate degree program on the Huntsville Campus.

The first six months of operation under NsG-381 made possible the increase of permanent staff from a total of 16 to 30 people; the second six-months saw an increase to 45 people; and at the end of this third reporting period, the permanent staff totaled 63 persons, including 10 employees jointly funded with the Univac Division of Sperry Rand Corporation. In addition, summer students increased from 7 in 1963 to 13 in 1964.

Considerable progress in facility development was made. The Research Institute building was officially completed on July 21, 1964; however, beneficial occupancy had been obtained on June 16, 1964. The UNIVAC 1107 system located in the Research Institute building became operational on July 3, 1964. By the end of the reporting period three laboratories were operational: Surface Physics, Electromagnetics, and Optics.

Research, mainly theoretical in nature, continued in 11 areas of the 13 originally outlined.

2.0 Resume of Research Efforts

Because of the support made possible through NsG-381, the Research Institute by the end of the reporting period had 16 persons in academic rank and 7 in full-time research positions. Only persons employed full time by the University of Alabama are counted in the foregoing. However, several are employed partly in the instructional program and partly by the Research Institute. The staff, with the participation of two faculty members from the Tuscaloosa campus, was engaged in a total of 14 research project areas counting both contract and grant supported studies. The research supported by NsG-381 represents 11 of the 13 areas anticipated in the proposal to NASA, dated August 31, 1962 (revised December 12, 1964), pages 20-24, areas A-M, as fields in which the staff of the Research Institute would work.

An outline of the research tasks for the year beginning March 1, 1964 is attached as Appendix A. The project areas are as follows: Theoretical Surface Physics, Theoretical Fluid Mechanics, Low Density Gas Dynamics, Hypersonic High-Temperature Gas Flow, High-Temperature Thermodynamics and Plasma Technology, Electromagnetics and Plasma Physics, Communications, Dielectrics, Control Theory, Operations Research, Structural Mechanics, Mathematics, Experimental Physics in Optics and Dielectrics, and Chemical Physics.

2.1 Problems Studied - Fourteen members of the staff of the Research Institute made presentations on research activities to Mr. William E. Scott, Research Program Officer, Grants and Research Contracts, National Aeronautics and Space Administration, on the occasion of his visit to the Research Institute on June 2, 1964.

Summaries of problems studied by the grant supported research are incorporated as Appendices B through L. Each Appendix identifies a project, names the principal research personnel pursuing the study and preparing the summary, and contains the references and figures which pertain to it. The appendices of project summaries are as follows:

"Effect of Gas-Surface Interaction Potential on Energy and Momentum Transfer in High Knudsen Number Gas Flow, Particularly for Condensable Gas Media" (A. B. Huang) Appendix B.

"Closed Invariant Subsets of Enveloping Transformation Groups" (H. C. Wasserman under the direction of H. Chu) Appendix C.

"An Elementary Problem of Numbers" (H. Chu and P. A. Lucas) Appendix D.

"Studies on the Influence of Outer Stream Vorticity on Steady and Unsteady Laminar Flows of the Boundary Layer Type" (A. A. Hayday and R. McGraw) Appendix E.

"Similar Flows About Axisymmetric Bodies Rotating in a Fluid at Rest" (A. A. Hayday) Appendix F.

"Studies on Axioms for Heterogeneous Continua" (A. A. Hayday) Appendix G.

"Studies in Hypersonic Flows" (R. Hermann and J. Thoenes) Appendix H.

"Groove Guide Measurements" (F. J. Tischer and F. W. Someroski) Appendix I.

"Projects in Mathematical Theory of Automatic Control" (C. D. Johnson) Appendix J.

"Pulse Modulated System with Adaptive Threshold (R. J. Polge and F. J. Tischer) Appendix K.

"Measurement of Orbital Parameters" (O. R. Ainsworth, C. M. Chambers, Jr. and F. J. Tischer) Appendix L.

2.2 Cooperation in University Research Activities - As indicated elsewhere in this report and as emphasized in the proposal on which grant NsG-381 was made, the concept on which the Research Institute was established is that it is an integral part, both academically and administratively, of the University of Alabama. A major purpose of the Research Institute is to contribute to the development of the entire University as a distinguished center of instruction and research in the aerospace physical sciences and engineering. Although the Research Institute has been in existence only a short time, its meaningfulness to research on the Tuscaloosa Campus and the instructional program on the Huntsville Campus is significantly manifest in terms of achieving this major objective.

In the case of three contracts, work was divided between the staff of the Research Institute and the main campus faculty members. Consultation and exchange of technical information relating to all contract research between main campus faculty and the professional staff of the Research Institute are frequent.

3.0 Research Related Educational Activities

The philosophy has been continued that senior personnel directing and supervising research should hold professorial appointments and participate in the formal classroom instruction. Conversely, the individual whose primary responsibility is teaching should also do research. The goal is to continuously advance the qualifications of the men who advise the graduate students. Such training takes place both in the classroom and on the research project.

The pursuit of learning generally, and the efforts of the professors and students specifically, are enriched and stimulated by the free exchange of ideas on as personal a basis as practicable. The Research Institute has sought to foster this by designing and supporting scientific meetings, lectures and seminars, and the publication of technical papers and research reports.

3.1 Formal Instruction and Thesis Supervision - Twelve members of the Research Institute staff hold appointments in academic departments on the main campus and eleven of them divide their time between research and instruction in the graduate and undergraduate program on the Huntsville Campus. Three members of the permanent faculty from the main campus had one-year teaching and research appointments to work in the Huntsville graduate program and for the Research Institute. It is contemplated that this type of arrangement between Huntsville and Tuscaloosa will be increased as the Research Institute grows.

During the past year 49 graduate courses were taught in Huntsville by Research Institute personnel. All but one of the University full time graduate faculty on the Huntsville Campus in physics, mathematics and engineering are formally spending part of their time in research and part in teaching. Research and teaching staff members are paid from grant and contract funds and/or teaching budget, appropriately determined by the distribution of their effort.

The interweaving of research and teaching activities adds variety and depth to both programs. Additionally, the dual opportunities in teaching and research aids in the recruiting and retention of qualified personnel.

A significant addition to the University's program in Huntsville was the announcement on May 12, 1964 by President Rose of the establishment of a regular four-year undergraduate resident program with majors in English, history, mathematics and physics. 194 full time students have enrolled in the fall quarter of 1964 in this program.

The Huntsville operation has now been officially designated as The University of Alabama, Huntsville Campus, to further emphasize the relationship of the Research Institute and the instructional programs in Huntsville with the Tuscaloosa Campus. The sheet, "Facts for Prospective Students" is attached as Appendix N.

At the writing of this report enrollment for fall 1964 is underway. Already there are 1547 enrolled in the undergraduate program, plus 774 students enrolled in the graduate program. The majority of the graduate students work in the aerospace industries and government agencies in the local area, and attend class part time. When adequate facilities are available at the Huntsville Campus, many of these students may devote full time to their studies in order to hasten their training.

In the fall quarter 1963, the resident master's degree program was established in Huntsville. M. S. degrees can be earned locally in mathematics, physics, and engineering. In engineering, a major is permitted in mechanical engineering, electrical engineering, and engineering mechanics with three options--solid mechanics, fluid mechanics, and dynamics and vibrations. Beginning in the fall quarter 1964, industrial engineering is added to the resident master's degree program. The opportunity to utilize the Research Institute's staff for instruction and for students to do significant research for their theses in the Research Institute was an essential factor in the establishment of the resident master's degree program.

Additional courses are available in aerospace engineering, chemical engineering, and metallurgical engineering.

3.2 Seminars and Special Lectures at the Research Institute - The Research Institute has continued the policy of presenting special lectures. The technical community is invited by circular to attend. Typical attendance is 40 engineers and scientists representing Marshall Space Flight Center of NASA, the Army Missile Command and the missile related industries in Huntsville. Attendance has been as high as 100. The dates, speakers and topics presented in the past six months follow

DATE	SPEAKER	TOPIC
April 10, 1964	Dr. W. C. Meecham Prof. of Fluid Mechanics University of Minnesota Minneapolis, Minnesota	Noise from Turbulent Boundary Layers
April 17, 1964	Dr. E. R. G. Eckert Prof. of Mech. Engr. & Director of Heat Trans Lab. University of Minnesota, Minneapolis, Minnesota	Thermodynamic Coupling of Heat and Mass Transfer
June 22, 1964	Dr. Mahlon C. Smith Project Engineer Bendix Corporation Ann Arbor, Michigan	The Effect of Free Stream Turbulence on the Laminar Boundary Layer Heat Transfer of Flat Plates and Circular Cylinders In Incompressible Flow.
June 30, 1964	Dr. R. J. Polge Asst. Professor of Elec. Engr. University of Ala., Huntsville	Communications Coding Theory
July 21, 1964	Mr. Martin E. Barzelay Prof. of Aero. Engineering Syracuse University (On leave to Harvard University)	Total Thermal Radiation Emitted from the Poisuelle Flow of An Atmospheric Pressure Argon Plasma Arc
August 3, 1964	Dr. J. J. Brainerd Staff Scientist, Astronautics Division of General Dynamics San Diego, California	Non-Equilibrium Hypersonic Nozzle Flows
August 4, 1964	Mr. J. Leith Potter Mgr. Research Branch Von Karman Gas Dynamics Fac. ARO, Inc. Tullahoma, Tennessee	Remarks on Experimental and Theoretical Research in Rarefied High-Speed Flows at VKF

DATE	SPEAKER	TOPIC
August 20, 1964	Dr. R. L. Causey Research Scientist Lockheed Missiles & Space Co. Palo Alto, California	On Closest Normal Matrices
August 31, 1964	Mr. Richard D. Wood Research & Opns. Engr. Jet Propulsion Lab. Pasadena, California	Gasdynamics Problems Associated with Entry into Planetary Atmospheres

3.3 Publications and Presentations of Papers

3.3.1 During the six months report period the following papers were presented at scientific meetings:

1. F. J. Tischer, "Waveguide with Arbitrary Cross-Section Considered by Conformal Mapping", presented at the Union Radio Scientific International in Washington, D. C., April 14, 1964.

2. C. D. Johnson, "A Problem of Letov in Optimal Control", presented at the Fifth Joint Automatic Control Conference, Stanford, California, June 25, 1964.

3. F. J. Tischer, "Groove Guide Measurements", presented at the 1964 Western Electronic Show and Convention, Los Angeles, California, August 26, 1964.

3.3.2 Published Articles - During the six months period four articles were published:

1. W. R. Garrett and R. A. Mann, "Elastic Scattering of Slow Electrons from Alkali Atoms", Physical Review, Vol. 135, A580 (1964), August 3, 1964

2. C. D. Johnson, "Optimal Bang-Bang Control with Quadratic Performance Index", in collaboration with W. M. Wonham, A. S. M. E. Transactions, Journal of Basic Engineering (March 1964).

3. C. D. Johnson and W. M. Wonham, "A Note on the Transformation to Canonical (Phase-Variable) Form", IEEE Transactions P. T. G. on Automatic Control, July, 1964.

4. C. D. Johnson and W. M. Wonham, "On a Problem of Letov in Optimal Control", Proceedings, Fifth Joint Automatic Control Conference, Stanford, California, June, 1964.

3.3.3 Papers submitted for publication during the six months period:

1. C. D. Johnson, "Optimal Control with Quadratic Performance Index and Fixed Terminal Time", in collaboration with J. E. Gibson, IEEE Transactions, P. T. G. on Automatic Control, October, 1964.

2. A. A. Hayday, "On Heat Transfer from Isothermal and Non-Isothermal Spinning Bodies of Revolution", to appear in the Journal of Heat Transfer.

3. A. A. Hayday, "Effects of Suction and Blowing on Plane Laminar Rotational Flows Near a Stagnation Point".

4. C. D. Johnson, "Singular Solutions in Problems of Optimal Control", Advances in Control Systems: Theory and Application, Vol. II, Chapter 6, edited by C. T. Leondes, published by Academic Press.

5. C. D. Johnson, "On a Problem of Letov in Optimal Control", ASME Transactions, Journal of Basic Engineering, March, 1965.

6. Hsin Chu, "Fixed Points in a Transformation Group", Pacific Journal of Mathematics.

7. Hsin Chu, "A Note on Transformation Groups With a Fixed End Point", Proceedings of the American Mathematical Society.

8. Hsin Chu, "Some Inheritance Theorems in Topological Dynamics", Journal of London Mathematical Society.

9. Hsin Chu, "An Elementary Problem on Numbers", in collaboration with Miss Patricia Lucas, The Pentagon, Mathematics Journal of Pi Mu Epsilon.

3.3.4 Laboratory Reports - During the six months period, five reports were issued:

U.A.R.I. Research Report No. 4, "Normal and Oblique Shock Wave Parameters for Various Dissociated Upstream Conditions in Air With Frozen Composition Across the Shock", J. Yalamanchili and J. Thoenes, July, 1964.

U.A.R.I. Research Report No. 10, "Effect of Gas-Surface Interaction Potential on Energy and Momentum Transfer in High Knudsen Gas Flow, Particularly for Condensable Gas Media", B. Huang, August, 1964.

U.A.R.I. Research Report No. 15, "An Evaluation of the Design, and a Theoretical Study of the Simulation Capability of an Eight-Megawatt Arc-Heated Hypersonic Wind Tunnel Facility", F. R. Winter, J. Yalamanchili, J. C. Dowdle, March, 1964.

U.A.R.I. Research Report No. 16, "Parabolic and Elliptic Waveguides Considered by Conformal Mapping", F. J. Tischer, H. Y. Yee, May, 1964.

U.A.R.I. Research Report No. 17, "Groove Guide Measurements", F. J. Tischer, F. W. Someroski, August, 1964.

3.4 Service on National Advisory Groups - Two members of the Research Institute continue to serve as advisers of national groups. Dr. Rudolf Hermann, Director, is a consultant to the National Science Foundation as a member of the Advisory Panel for Engineering, Division of Mathematical, Physical and Engineering Sciences. Dr. F. J. Tischer, Assistant Director is a member of the Research Advisory Committee on Communications, Instrumentation, and Data Processing. The purpose of this committee is to advise the NASA in the formulation of its programs of research in the indicated areas.

4.0 Development of Capability

The Research Institute has continued to increase its capability to do research as evidenced by the completion of construction of its first laboratory-office building, by the acquisition of specialized instrumentation, by the increase in staff, and by obtaining additional contract and specific grant support.

4.1 Facilities - Previous reports have given the design philosophy and space allocation in the 64,000 square foot first phase laboratory-office building. Two ground views taken on August 26, 1964 are presented. Figure 1 shows the two-story main wing in the background and the single story C and D wings in the foreground. Figure 2 shows A and B wings in the foreground. The roads, parking lots, and landscaping have not been completed, but temporary access and parking are available. Completion of all outside work is expected by November 1, 1964. The majority of the roads, drainage (storm and sanitary sewers), parking lots and earth moving work and materials are being donated by the city, county, and state. The civil engineer's estimate for this construction is $186,450.

The UNIVAC 1107 system was moved into the above building on June 18 and became operational July 3, 1964. It consists of 65,000 word core, two drums of 786,000 words each, and two satellite computers (UNIVAC 1004) on line. The system contains two high speed printers. The equipment for the data link to the main campus is on order.

Because the $3,000,000 State Bond Issue, which is furnishing the majority of capital items for the research, needs augmentation in order to achieve the facility's goal, a proposal for a Graduate Science Facilities Grant was prepared by the University and submitted on January 15, 1964 to the National Science Foundation in the amount of $332,697. The State of Alabama will match whatever funds are granted in response to this proposal.

The University on May 12, 1964, announced plans for a second instructional building in Huntsville. $500,000 has been allocated by the University. A fund raising drive in the north Alabama region, which has a goal of an additional $750,000, has met with enthusiastic response. By October 6, 1964, $633,000 had been pledged.

In the coming year space in the Research Institute will be used temporarily as class rooms for graduate classes beginning at 3:50 P. M. Use of the space will not hamper the development of research functions.

4.2 Manpower - The funds made available under NsG-381 have made possible a continuous increase in staff. Table I, Research Institute Personnel Statistics, has been prepared to show the number of persons employed at the end of six-month intervals over the past two years. For the interest of clarity we have listed the summer students separately. Staff has been recruited to the limits of funds available in NsG-381 for salaries. Contracts and specific grants recently awarded will permit hiring of additional personnel. We are maintaining the policy that in any new area a well qualified head of the group will be hired first. He, in turn, will establish a research program and help recruit members of his group.

Biographies of academic and full time research staff who have joined the Research Institute during this reporting period or who have accepted positions are included in Appendix M.

4.3 Contracts and Specific Grants - During the period of this report contracts and specific grants have more than doubled in number from 4 to 11, and in amount from $204,379.80 to $480,309.86. On August 31, 1964 there were 3 contracts and one specific grant (consisting of two major tasks) with the U. S. Army, totaling $214,275.00, and there were 6 contracts and one specific grant with the National Aeronautics and Space Administration (not counting NsG-381), totaling $266,034.86.

5.0 Financial Report

The financial report of funds spent from this grant is provided by the Comptroller of the University and attached as Tables 2 and 3. As shown by Table 2, actual expenditures at the Research Institute were $184,532.84 ($185,462.66 minus $929.82 at Tuscaloosa Campus) or 52% of the amount available for the grant year ($354,941). The rate of expenditure exceeds 50% because expenditures for equipment in the amount of $53,211.25 was slightly larger than average during this period. Those figures show that during this grant year, the Research Institute is using the available funds to their limit in order to grow.

It is noted that direct student salaries increased from $15,203.38 in the second six months period to $21,254.90 ($22,004.90 minus $750.00 at Tuscaloosa Campus) in this third period, emphasizing the student participation in research.

Administrative expenses during the period amounted to $73,301.46 of which $42,296.40 was provided by overhead on NASA Grant NsG-381.

6.0 Outlook

Although the Research Institute has not yet reached a size in staff and research funding where its operation is self-sustaining, the outlook is better than it has been anytime since its establishment. The rapid buildup of the instructional program described in paragraph 3.1, and the continued movement of aerospace oriented companies into the Research Park, adjacent to the Research Institute, will have a favorable effect on the development of the Research Institute. Although it is expected that governmental agencies will be the prime source of research grants and contracts for the Research Institute, already three private companies have made overtures for a team effort in research. These latter will be accepted only when it can be expected that the academic character of the Research Institute will be preserved.

In the past, because of lack of a suitable building, much effort went into planning and equipping the building and laboratories. The equipping of the laboratories will continue, but it is expected that the Institute will enter a new phase: the strong theoretical effort will continue, but will be supplemented by an increasing amount of experimental research.

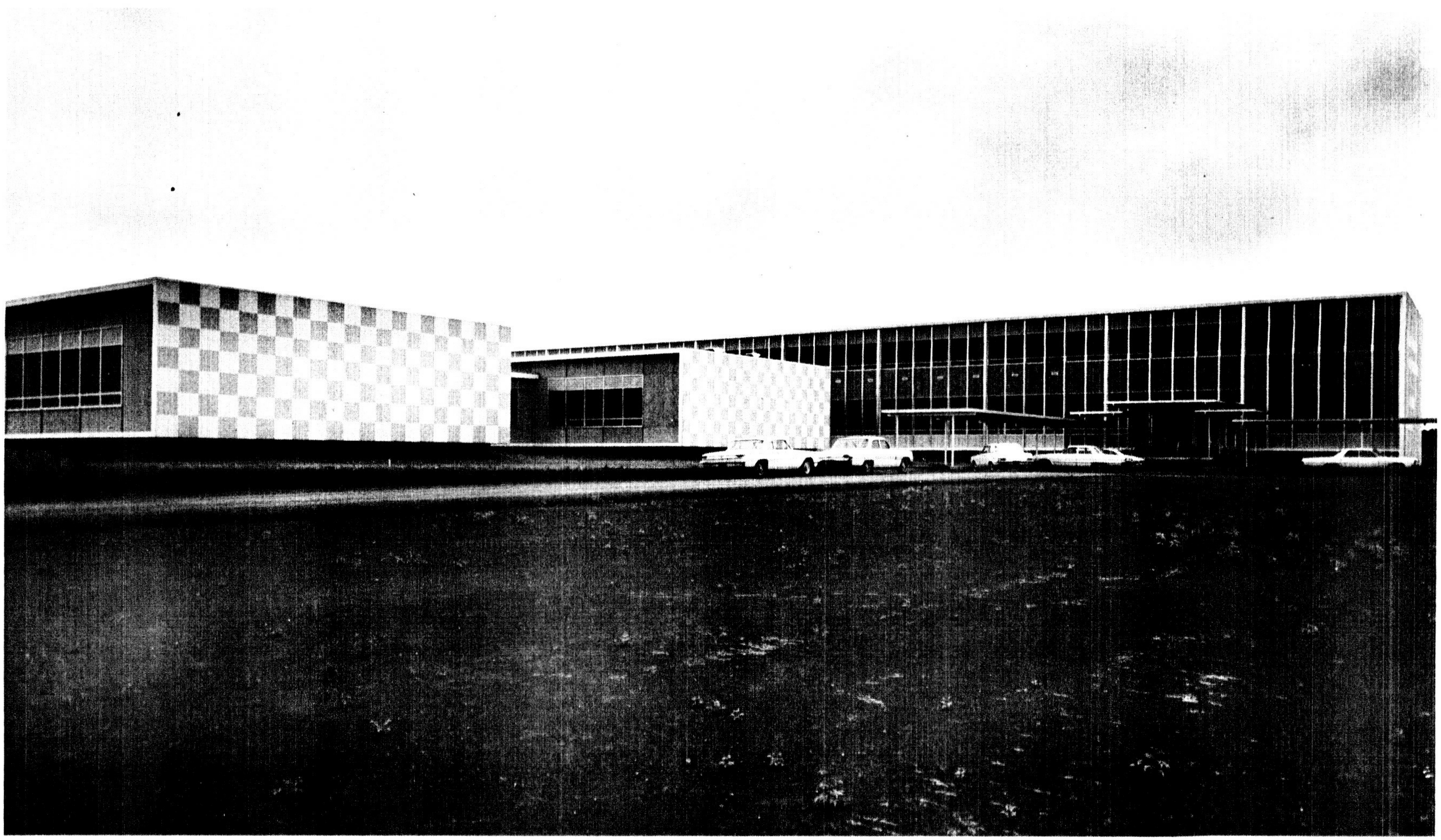

FIGURE 1

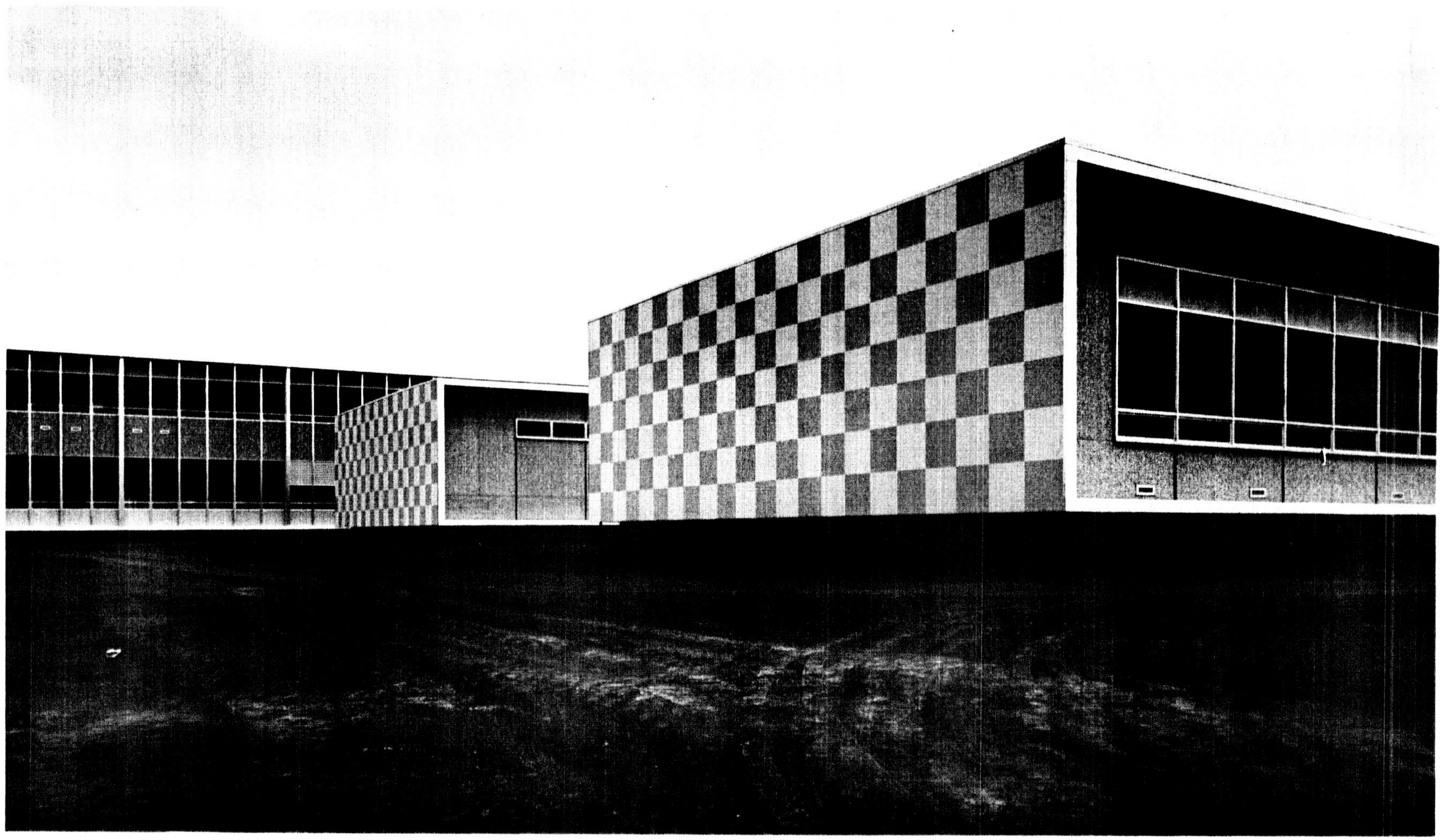

FIGURE 2

TABLE I

RESEARCH INSTITUTE PERSONNEL STATISTICS

A. Number of Permanent Staff, Including Part Time

	8/31/62	2/29/63	8/31/63	2/29/64	8/31/64
Academic	2	6	9	15	16
Permanent Research	0	0	5*	6*	16**
Administration	4	6	8	13	17
Technical Service	2	2	2	2	4
Research Assistants (Graduate)	1	2	4	7	8
Undergraduate Assistants	0	0	2	2	2
Totals	9	16	30*	45*	63**

* Includes 1 person jointly funded with the Univac Division of Sperry Rand Corporation.

** Includes 10 persons jointly funded with Univac.

B. Number of Summer Students

	1963	1964
Research Assistants (Graduates)	1	3
Undergraduate Assistants	6	10
Totals	7	13

TABLE 2

UNIVERSITY OF ALABAMA RESEARCH INSTITUTE
Semi-Annual Statement of Expenditures and Encumbrances
For Period March 1, 1964, through August 31, 1964
NASA Research Grant NsG-381
and Supplement No. 1

Salaries		
Professional	$44,098.40	
Student	22,004.90	
Supporting Services	16,277.06	$ 82,380.36
Supplies		1,362.47
Travel and Communication		5,034.00
General Expense		6,413.04
Equipment		53,211.25
Total Expenditures		$148,401.12
Overhead 25% of Direct Costs ($147,626.27)		36,906.57
Overhead 20% of Direct Costs ($ 774.85)*		154.97
		$185,462.66
Plus Encumbrances		
Equipment on Order		21,559.30
Overhead 25% on Encumbered Items		5,389.83
Total Expenditures and Encumbrances		$212,411.79

* This amount expended during this period for
Supplement No. 1 - Chemistry - Tuscaloosa Campus

Salaries, Student	$750.00
Supplies	24.85
	$774.85
Overhead 20% of Direct Costs	154.97
	$929.82

I certify that this statement of expenditures and encumbrances is correct and in accordance with the terms of the grant.

Luther C. Callahan
Comptroller

TABLE 3

UNIVERSITY OF ALABAMA RESEARCH INSTITUTE
Semi-Annual Statement of Expenditures
For Period March 1, 1964, through August 31, 1964
(Administrative Portion)

Salaries		
Administrative	$52,002.48	
General Labor	2,437.90	$54,440.38
Supplies		2,776.50
Communications		5,358.22
Travel		1,361.68
General Expense		9,293.52
Equipment		71.16
Total		$73,301.46

NOTE: These expenditures are provided for by the $42,296.40 in overhead on NASA Grant NsG-381 for this period and overhead received on other contracts and grants which the University of Alabama Research Institute has with governmental agencies.

I certify that this statement of expenditures is correct.

Luther C. Callahan
Comptroller

APPENDIX A

RESEARCH TASKS ON NsG-381 FOR THE SECOND GRANT YEAR
31 August 1964

A. Theoretical Surface Physics

Dr. R. A. Mann, Associate Professor of Physics and Dr. W. R. Garrett, Assistant Professor, have been working in the general areas of low energy charged partical scattering and orbital mechanics. They will carry out studies involving the scattering of low energy protons and electrons by atoms and ions. During the course of previous work, they have developed computational methods for obtaining atomic and ionic charge distributions that will be useful in the planned work involving scattering calculations. They also plan to use their atomic structure programs as a first step toward setting up the problem of the interaction of an intense beam of coherent light with atoms. Their other activities include consideration of various processes which may occur in space and which may be of interest in astrophysics.

B. Theoretical Fluid Mechanics

Dr. A. A. Hayday, Associate Professor of Engineering Mechanics and Mathematics, in his remaining time with the Research Institute through December, 1964, will work to complete current projects. Several papers whose contents range from exact work on boundary layer theory to certain aspects of the mathematical structures of heterogeneous continua have been recently submitted for publication. The papers which remain to be completed are on second-order boundary layer theory with applications to problems on boundary layer control, ablation phenomena, and magneto-hydrodynamics. If time permits, two problems on combustion and thermodynamic cross-phenomena shall also be finished.

C. Low Density Gas Dynamics

Dr. A. Ben Huang, Assistant Professor in Mechanical Engineering, prior to his resignation on September 1, 1964, was investigating the molecular surface interaction in the free molecule and transition flow regions, which govern the transfer phenomena in the high-altitude aerodynamics. Continuation of research in low density gas dynamics is contingent upon success in recruiting a scientist with qualifications and interests in this area.

D. Hypersonic High-Temperature Gas Flow

Mr. Jurgen Thoenes, Research Assistant and Ph. D. Candidate, is continuing to work under the direction of Dr. R. Hermann, on the problem of hypersonic flow around blunt bodies and pointed cones with non-equilibrium dissociation. Previous calculations, using the direct method, on flow around a cylinder with oxygen dissociation, reaching up to the sonic line, are extended beyond into the supersonic regime. The non-equilibrium flow fields around very large bodies will be investigated for the convergency to the equilibrium case. The studies are being extended from the two-dimensional problem of a cylinder to a spherical body. Afterwards, the nitrogen dissociation will be taken into account for all those flow problems. Recently, the flow equations around pointed cones under the same hypersonic non-equilibrium conditions are formulated for numerical solution.

E. High-Temperature Thermodynamics and Plasma Technology

Upon recruitment, a successor to Dr. E. R. F. Winter, Associate Professor of Chemical Engineering, will continue theoretical and experimental research in the areas of high-temperature thermodynamics, in particular research in thermodynamic properties of, and energy and mass transfer to ionized gases in plasma generators. The required operating characteristics of the power conversion and control equipment (300 KW) for this laboratory has already been developed in coordination with a possible manufacturer. It will be installed along with the first arc and required instrumentation, thereby enabling experimental research to be done prior to March, 1965.

F. Electromagnetic and Plasma Physics

Dr. F. J. Tischer, Professor of Electrical Engineering and Assistant Director of the Research Institute, is carrying out and directing research in electromagnetic wave propagation, space communications, and plasma physics. He and his associates are studying theoretically wave propagation phenomena encountered at re-entry into and at the powered flight of space vehicles in planetary atmospheres; theoretically and experimentally the transmission and detection of millimeter waves; and the coherence, polarization, and non-linear phenomena at sub-millimeter and optical wave lengths are other topics to be investigated. In the area of communications, the problems of communication channels with memory are under investigation (see G);

a model for the optimization of tracking and guidance operations of vehicle-born systems; and the effects of quantization on the information transmitted at optical frequencies are being studied. Studies in plasma orbit theory and of self-supporting plasma structures in inter-planetary space are topics to be investigated in the future.

Some of the planned laboratories will be equipped, instrumented and activated during the present year. Besides Dr. Tischer, other principal people engaged in these studies are Dr. O. Ainsworth, Professor of Mathematics, Dr. N. F. Audeh, Visiting Associate Professor of Electrical Engineering, Dr. Charles Chambers, Dr. F. H. Mitchell, Jr., Dr. R. Polge, Assistant Professor of Electrical Engineering, and Mr. H. Y. Yee.

G. Communications

Dr. R. Polge, Associate Professor of Electrical Engineering, is presently interested in the field of communication theory, specifically in pulse code modulation. In standard pulse code modulation, the signal is compared to a fixed threshold. When a pulse (or one) is sent, an error results if the sum of the pulse and noise received is less than the threshold level. Similarly, when no pulse (or zero) is sent, an error results if the noise received is larger than the threshold level.

He intends to continue to carry out and direct research using an adoptive scheme with memory, whereby the noise can be partially predicted and the threshold varied accordingly, thus reducing the probability of error. The research is both theoretical and experimental. Dr. Polge is planning to continue also research in statistical design and sampled-data systems for applications in communication and control theory.

H. Control Theory

Dr. C. D. Johnson, Associate Professor of Electrical Engineering, has a major interest in the mathematical theory of automatic control. He will employ the Hamilton-Jacobi equation and the theory of optimal control to study optimal guidance and to control schemes for stationary linear systems with quadratic indices of performance, bounded control, and fixed terminal time. Further research plans include: (i) a study of Game Theoretic problems in optimal control, (ii) a study of the role of singular solutions in problems of trajectory optimization, and (iii) the development of a more general theory for optimization problems with bounded state variables.

Mr. C. F. Chen, formerly a Professor of Electrical Engineering at Christian Brothers College, has been hired as a Research Associate for the Control Sciences Laboratory being built up by Dr. Johnson. He will join the Research Institute on 1 November 1964, and will be conducting research in Laponov's Stability Theory as applied to control systems.

I. Operations Research

It is planned to hire a senior level person (Ph.D. mandatory) to organize the nucleus of an Operations Research and Systems Analysis Group. He will develop techniques to mathematically describe the interrelationship of the factors which affect a given task, will simulate elements of the task by analytic and/or numerical operations to arrive at a suitably defined optimum method of accomplishing the task.

J. Structural Mechanics

Dr. W. Kubitza has been hired as Professor of Engineering Mechanics and Head of the Structural Mechanics Laboratory. He will report in the latter part of the period (February 1, 1965) and will begin research in structural mechanics by assembling the apparatus and equipment needed to study the distribution of strain in structures. Presently, he is interested in the use of optical methods for this purpose. Participating in theoretical studies of structural mechanics, starting September 1964, will be Dr. G. Wempner, Professor of Engineering Mechanics.

K. Mathematics

Under the direction of Dr. Hsin Chu, Associate Professor of Mathematics, the Institute plans to continue and extend the mathematics research work of the past year. In the field of pure mathematics, among other things, we study the Lie theory of transformation groups, Liapunov's method, Pontryagin principle, differential equations, and game theory. In the field of applied mathematics we concentrate our efforts on numerical calculus in general and numerical solution of differential equations in particular.

L. Experimental Physics in Optics and Dielectrics

Dr. J. H. Kallweit, Senior Research Associate, is presently concerned with the instrumentation of the optical laboratory. The laboratory facility will be used for the optical sensing and measurement of such phenomena as are associated with high-temperature thermodynamic experiments dealing with electric arcs in various types of gases.

Much of the equipment has been ordered and it is anticipated that the optical facility will be completed this year.

In the interim, while this equipment is being assembled, Dr. Kallweit is studying many of the problems in the areas in which he is planning to direct research. He is planning to utilize the equipment to provide the necessary laboratory environment to permit investigation into such problems as: detection of reflected light on various types of surfaces, the interferometric evaluation of light sources, and electrical behavior of dielectrics at boundary surfaces.

M. Chemical Physics

Dr. W. F. Arendale is joining the Research Institute (September 1, 1964) as Professor of Chemistry and as an Assistant Director. He plans to work in areas of chemical physics. Emphasis will be placed on describing material properties that may be understood by studies of the propert ies of individual molecules and atoms. The behavior of molecules in the highly excited states, as may be obtained under conditions generated by the plasma and arc equipment availalbe at the Research Institute, will be studied. The chemical reactivity of these molecules will be of particular interest.

APPENDIX B

EFFECT OF GAS-SURFACE INTERACTION POTENTIAL ON ENERGY AND MOMENTUM TRANSFER IN HIGH KNUDSEN NUMBER GAS FLOW, PARTICULARLY FOR CONDENSABLE GAS MEDIA

by A. B. Huang

As the trajectories of space probes and earth satellites become more precise, the transfer processes in the free-molecule flow regime take on added significance. When the characteristic dimension of the space probe is many times smaller than the molecular mean free path in free space, the Knudsen number K 1, the flow phenomena are determined almost entirely by the collisions with the surface and are practically unaffected by the inter-molecular collisions. A large number of investigations of the free molecule flow regime have been reported in the literature. A great majority of investigations were conducted under the assumption of known accommodation coefficients. Unfortunately, the applicability of available values of the thermal accommodation coefficient to the calculation of energy and momentum transfer in the free molecular flow regime is quite uncertain. Furthermore, the majority of the investigators failed to grasp the extreme importance of clearly defining the surface conditions, resulting in a serious deficiency in many of the reported results.

In the phenomenon of the interaction between gas molecules and a solid surface, three phases can be considered:

(1) Incidence of gas molecules on the surface.
(2) Accommodation process of gas molecules on the surface.
(3) Re-emission of gas molecules from the surface.

The first phase is well understood. The objective of the report UARI, No. 10, now being distributed, is to present a theoretical treatment on the second and third phases and the application of the results to the aerodynamics calculation of energy and momentum transfer between gas and surface in the free molecule flow regime. The study concerned kinetics of molecules in a free molecular flow regime under influence of strong interactions with the surface (the physical nature of gas and solid atoms). The Kirkwood-Muller potential has been used for the gas-surface interaction. Considerations include physical adsorption at the surface, energy and momentum transfer between molecules and solid atoms. The distribution function of the trapped diffusely re-emitted molecules is derived. A new method which makes reliable aerodynamic calculations on energy transfer without knowing the accommodation coefficient was presented.

The new method was checked with Stalder and Jukoff's calculations. The comparison between the results of the present theory and Stalder's calculation showed that the present theory offers more reliable results with respect to a number of gases and solids. The significance of adsorbed layer in free molecule flow regime is explored. Aerodynamic contribution to drag on a body in a free molecule flow regime due to adsorption at the surface was also considered.

APPENDIX C

CLOSED INVARIANT SUBSETS OF ENVELOPING TRANSFORMATION GROUPS

By H. C. Wasserman under direction of H. S. Chu

Section I.

Definition 1. Let (X, T) be a transformation group. Then (X, T) is phase-compact provided that X is compact Hausdorff.

Definition 2. Let (X, T) be a phase-compact transformation group. Then:

(i) The enveloping semigroup of (X, T), denoted by $E(X, T)$ or E_X or E, is defined to be the closure in the Cartesian product space X^X of the transition group G of (X, T). We provide E with the semigroup operation of functional composition.

(ii) The enveloping transformation group of (X, T) is defined to be the transformation group $(E(X, T), T)$ with action $(p, t) \to p \circ \pi^t$, $(p \in E(X, T), T \in T)$, where π is the action of (X, T) and $\circ$ denotes functional composition.

Definition 3. Let X be a topological space. Then:

(i) The component partition of X, denoted by $\mathcal{C}_X$ or $\mathcal{C}$, is defined to be the partition of X into its connected components.

(ii) For every $x \in X$, $x\mathcal{C}_X$ is defined to be the connected component of x in X.

LEMMA 1. Let (X, T) be a transformation group, and let $x \in X$. Then the following statements are pairwise equivalent:

(i) $x\mathcal{C}$ is invariant

(ii) $xT \subset x\mathcal{C}$

(iii) $\overline{xT} \subset x\mathcal{C}$

PROOF: Statements (ii) and (iii) are equivalent since $x\mathcal{C}$ is closed.

Assume (i). We prove (ii):

For every $t \in T$, $(x\,t)\mathcal{C} = x\mathcal{C}t \subset x\mathcal{C}$, and hence $(x\,t)\mathcal{C} = x\mathcal{C}$.

Thus $x\mathbf{T} \subset x\mathcal{C}\mathbf{T} = \bigcup_{t \in T} x\mathcal{C}t = \bigcup_{t \in T} x\,t\mathcal{C} = x\mathcal{C}$.

Assume (ii). We prove (i):

For every $t \in T$, $x\,T = x\,T\,t \subset x\mathcal{C}t = x\,t\mathcal{C}$.

Hence $x\mathcal{C}\,t = \bigcup_{t \in T} x\,t\mathcal{C}$ is connected.

Thus $x\mathcal{C}\,T = x\mathcal{C}$

<u>LEMMA 2</u>. Let (X, T) be a transformation group with T connected. Then for every $x \in X$, $x\mathcal{C}$ is invariant.

<u>PROOF</u>: For every $x \in X$, $x\,T$ is connected and $x \in x\,T$, and hence $x\,T \subset x\mathcal{C}$. Thus, for every $x \in X$, $x\mathcal{C}$ is invariant (by Lemma 2).

<u>REMARK</u>: Let (X, T) be a phase-compact transformation group with T connected, and let x, y be proximal elements of X. Then $x\mathcal{C} = y\mathcal{C}$.

<u>PROOF</u>: For every index α of X, $(xT \times yT) \cap \alpha \neq \emptyset$, and hence $(\overline{xT} \times \overline{yT}) \cap \alpha \neq \emptyset$.

Then $(\overline{xT} \times \overline{yT}) \cap \Delta_X \neq \emptyset$, and hence $\overline{xT} \cap \overline{yT} \neq \emptyset$. But $\overline{xT}$ and $\overline{yT}$ are connected.

Thus $x\mathcal{C} = y\mathcal{C}$.

<u>LEMMA 3</u>. Let X be a compact Hausdorff space. Then $\mathcal{C}$ is a star-closed decomposition of X.

<u>PROOF</u>: Clearly $\mathcal{C}$ is a decomposition of X.

To show that $\mathcal{C}$ is star-closed, it suffices, by 1.36 of Coll. Vol., to show that for every $x \in X$ and for every neighborhood U of $x\mathcal{C}$, there exists a neighborhood V of x with $V\mathcal{C} \subset U$, where $V\mathcal{C} = \bigcup_{y \in V} y\mathcal{C}$.

To this end, let $x \in X$, and let U be a neighborhood of $x\mathcal{C}$. Now,

$x\mathcal{C}$ is the intersection of all open-closed sets in X which contain $\{x\}$ The collection of all such sets is a closed filter base with adherence $x\mathcal{C}$.

Hence there is an open-closed set V in X with $x \in V$ and $V \subset U$. For every $y \in V$, $y\mathcal{C} \subset V$, and hence $V\mathcal{C} \subset V \subset U$. Thus $\mathcal{C}$ is star-closed.

<u>Theorem</u>: Let (X, T) be a phase-cpt transformation group, and let T be connected. Then:

(i) Every member of $\mathcal{C}$ is a minimal orbit-closure if and only if $\mathcal{C} = X/T$, where $X/T = \{\overline{xT} \mid x \in X\}$.

(ii) If every member of $\mathcal{C}$ is minimal, then X/T is a star-closed decomposition of X.

(iii) If every member of $\mathcal{C}$ is minimal, then (X, T) is weakly almost periodic.

<u>PROOF</u>:

(i) Suppose every member of $\mathcal{C}$ is minimal. Then $\mathcal{C} \subset X/T$. Moreover, for every $x \in X$, $\overline{xT} \subset x\mathcal{C}$ (since $\overline{xT}$ is connected) and hence $\overline{xT} = x\mathcal{C}$ (since $x\mathcal{C}$ is minimal). Thus $\mathcal{C} = X/T$.

Suppose, conversely, that $\mathcal{C} = X/T$. Then X/T is a partition, and thus every member of $\mathcal{C} = X/T$ is minimal (by 2.23 of Coll. Vol.).

Statement (ii) is immediate from (i) and Lemma 3. Statement (iii) is immediate from (ii) and 4.24 of Coll. Vol.

<u>COROLLARY</u>: Let (X, T) be a phase-compact transformation group such that T is connected and every member of $\mathcal{C}$ is minimal. Then for every open subset U of X, $U\mathcal{C} = UT$, and hence $\mathcal{C}$ is star-open.

<u>PROOF</u>: $\mathcal{C} = X/T$ (by the preceding theorem). The conclusion follows from 2.29 of Coll. Vol.

Section II.

Standing Notation: Throughout this section, (Y, T) shall be a phase-compact transformation group, X shall be a nonvacuous closed invariant subset of $E = E(Y, T)$, I shall be the set of all idempotent elements of X, Π shall be the proximal relation in E, and $\mathcal{C}$ shall be the component partition of X.

REMARK 1. Let $u, v \in I$. Then $u \Pi v$ if and only if there exists $q \in X$ with $uq = vq$.

PROOF: It is known that $u \Pi v$ if and only if there exists $p \in E$ with $up = vp$ (see A).

For this latter condition to hold, it is necessary and sufficient that there exist $q \in X$ with $uq = vq$; the sufficiency is obvious, and the necessity follows by setting $q = up$ $(= vp)$.

REMARK 2: If T is connected, then $\mathcal{C} = \{u\mathcal{C} \mid u \in I\}$.

PROOF: For every $p \in X$, $p\mathcal{C}$ is nonvacuous closed invariant (by Lemma 2 of Section I). Then for every $p \in X$, there exists $u \in I \cap p\mathcal{C}$, whence $p\mathcal{C} = u\mathcal{C}$.

Thus $\mathcal{C} = \{u\mathcal{C} \mid u \in I\}$.

LEMMA 1. For every $u \in I$, $uE = uX$.

PROOF: For every $p \in X$, $pX \subset pE \subset X$.

Hence, for every $u \in I$, $uX \subset uE = u(uE) \subset uX$.

Thus, for every $u \in I$, $uE = uX$.

THEOREM 1. The following statements are valid:

(i) For every $p \in X$. $p\mathcal{C}$ is a nonvacuous closed invariant subset of X and hence of E.

A. : A Semigroup Associated with a Transformation Group, by R. Ellis, Trans. Amer. Math. Soc., Vol. 94 (1960) pp. 272-281

(ii) For all $p, q \in X$, if $p \Pi q$, then $p\mathcal{C} = q\mathcal{C}$. If, moreover, T is connected, then:

(iii) E is connected

(IV) Every member of $\mathcal{C}$ is minimal if and only if $\mathcal{C} = X/T$, where $X/T = \{\overline{pT} \mid p \in X\}$.

(V) If every member of $\mathcal{C}$ is minimal, then:

(a) X/T is a star-closed decomposition of X.

(b) (X, T) is weakly almost periodic.

(c) For every open subset U of X, $U\mathcal{C} = \bigcup T$, and hence $\mathcal{C}$ is star-open.

(d) For every $u \in I$, $u\mathcal{C} = u X$.

(e) For every $p \in X$, $p\mathcal{C} = p X$.

<u>PROOF</u>: Statement (i) follows from Lemma 2 of Section I, and Statement (ii) follows from the remark of Section I.

Suppose that T is connected. Then statement (iii) follows from the fact that $E = \overline{id_X T}$, and statement (IV) follows from the theorem of Section I.

To prove (V), suppose that every member of $\mathcal{C}$ is minimal.

Then statements (a) and (b) follow from the theorem of Section I, and statement (c) follows from the corollary to the theorem of Section I.

We show that statements (d) and (c) hold: We have that $\mathcal{C} = X/T$ (by statement (IV)). Hence, for every $u \in I$, $u\mathcal{C} = \overline{uT} = uE = uX$ (by Lemma 1). Hence (d) holds.

Let $p \in X$. Then $p X$ is nonvacuous closed invariant, and hence $u \in I \cap p X$.

Then $p\,X \subset p\,E$

$$= \overline{p\,T}$$

$$= p\mathcal{C} \qquad \text{(by statement (IV))}$$

$$= u\mathcal{C} \qquad \text{(since } u \in p\mathcal{C}\text{)}$$

$$= u\,X \qquad \text{(by statement (d))}$$

$$\subset p\,X \qquad \text{(since } u \in p\,X \text{ and } p\,X \text{ is a right ideal).}$$

Thus $p\,\mathcal{C} = p\,X$.

LEMMA 2. Let (B, S) be a phase-compact transformation group, and let A be a nonvacuous closed invariant subset of B. Then the following statements are equivalent:

(i) A is not minimal.

(ii) There exist $x, y \in A$, and there exists a continuous map φ of A into $\mathbb{R}\,[o, 1]$ with $x\,S\,\varphi = \{o\}$ and $y\,S\varphi = \{1\}$

PROOF:

(i) Implies (ii):

Suppose A is not minimal. Then there exists $x \in A$ with $A - \overline{xS} \neq \emptyset$. Let $y \in A - \overline{xS}$. Then $\overline{xT} \cap \overline{yT} = \emptyset$. Hence there exists a continuous map φ of A into $\mathbb{R}\,[o, 1]$ with $\overline{xS}\,\varphi = \{o\}$ and $\overline{yS} = \{1\}$.

Now (ii) implies (i) is evident since, by continuity of φ, $\overline{xS}\,\varphi = \{o\}$ and $\overline{yS} = \{1\}$.

LEMMA 3. Let (B, S) be a phase-compact transformation group, with S a compact separated group, and let A be a nonvacuous closed invariant subset of B. Then the following statements are pairwise equivalent:

(i) A is not minimal.

(ii) There exist $x, y \in A$, and there exists a continuous map φ of A into $\mathbb{R}\,[o, 1]$ with $x\,S\,\varphi = \{o\}$ and $y\,S = \{1\}$.

(iii) There exist x, $y \in A$, and there exists a continuous map Φ of A into $\mathbb{R}[o, 1]$ with $x\Phi = o$ and $y\Phi = 1$ and such that $z\Phi = z\,t\Phi$ for every $z \in A$ and for every $t \in S$.

PROOF: Statements (i) and (ii) are equivalent, by Lemma 2, and statement (iii) evidently implies statement (ii).

We show that (ii) implies (iii). Suppose there exist x, $y \in A$ and there exists a continuous map φ of A into $\mathbb{R}[o, 1]$ with $x S\varphi = \{o\}$ and $y S \quad = \{1\}$.

For every $t \in S$, $\pi^t\varphi$: $B \to \mathbb{R}[o, 1]$ is continuous, $x\pi^t\varphi = o$, and $y\pi^t\varphi = 1$, where π^t: $B \to B$ is defined by $z \to z\,t$ $(z \in B)$. For every $z \in A$, φ_z: $S \to \mathbb{R}[o, 1]$, defined by $t \to z\,t\varphi = z\ \pi^t\varphi$ $(t \in S)$, is continuous.

Let μ be a normalized left Haar measure in S. Then, for every $z \in A$, $\int t\,\varphi_z\,d\mu(t) \in [o, 1]$. Define Φ: $A \to \mathbb{R}[o, 1]$ by $z\Phi = \int t\,\varphi_z\,d\mu(t)$ $(z \in A)$. The map Φ is continuous. Moreover, $x\Phi = \int x\,t\varphi d\mu(t) = \int o\,d\mu(t) = o$, and $y\Phi = \int y\,t\varphi d\mu(t) = \int 1\,d\mu(t) = 1$. Finally, for every $z \in A$, and for every $s \in S$, we have that $zs\Phi = \int z\,s\,t\varphi d\mu(t) = \int z\,t\varphi\,d\,\mu(t) = z\Phi$.

LEMMA 4. Let (B, S) be a phase-compact transformation group, with S a compact separated group, and let A be a nonvacuous closed invariant subset of X. Then the following statements are pairwise equivalent:

(i) A is minimal.

(ii) For all x, $y \in A$, and for every continuous map φ of A into $\mathbb{R}[o, 1]$, either $x S\varphi \neq \{o\}$ or $y S\varphi \neq \{1\}$.

(iii) For all x, $y \in A$, and for every continuous map φ of A into $\mathbb{R}[o, 1]$, if both $x\varphi = o$ and $y\varphi = 1$, then there exists $z \in A$ and there exists $t \in S$ with $z\varphi \neq zt\varphi$.

PROOF: Immediate from Lemma 3 by logical inversion.

THEOREM 2. Let S be a compact separated subgroup of T (if such exists). Then the following statements are pairwise equivalent:

(i) (X, S) is minimal.

(ii) For all u, v ϵ I, and for every continuous map φ of X into $\mathbb{R}$ [o, 1], either $u\,\overline{S}\varphi \neq \{o\}$ or $v\,\overline{S}\varphi \neq \{1\}$.

(iii) For all u, v ϵ I, and for every continuous map φ of X into $\mathbb{R}$ [o, 1], if both $u\varphi = o$ and $v\varphi = 1$, then there exists p ϵ X and there exists t ϵ S with $p\varphi \neq p\,t\varphi$.

PROOF: (i) Implies (ii) by Lemma 4, and (ii) implies (iii) is evident. We show that (iii) implies (i).

Assume that (iii) holds, and just suppose that (X, S) is not minimal.. Then there exist p, q ϵ X, and there exists a continuous map φ of X into $\mathbb{R}$ [o, 1] such that $p\varphi = o$, $q\varphi = 1$, and $r\varphi = r\,t\varphi$ for every r ϵ X and t ϵ S (by Lemma 3).

Then $\overline{p\,S}\,\varphi = \{o\}$ and $\overline{q\,S}\,\varphi = \{1\}$. There exists $u \in I \cap \overline{p\,S}$ and there exists $v \in I \cap \overline{q\,S}$. Then $u\varphi = o$ and $v\varphi = 1$. Hence, by our assumption of (iii), there exists r ϵ X and t ϵ S with $r\varphi \neq r\,t\varphi$, a contradiction. Thus (X, S) is minimal.

APPENDIX D

AN ELEMENTARY PROBLEM ON NUMBERS

by Hsin Chu and P. A. Lucas[1]

1. Introduction

In this note we consider the following problem: "Let T_o be an unknown number of objects such that no object can be divided into a fractional part. If $^p/r$ parts of T_o plus $^s/r$ of one object are removed from T_o, where $0 < s < r$ and $0 < p < r$, the remainder, T_1, has no fraction. If the process is continued n times so that $^p/r$ parts of T_k, plus $^s/r$ of one object are removed from T_k, leaving a remainder, T_{k+1} $(k=0,1,2, \ldots, n-1)$, which has no fractional parts, the last remainder, T_n, will be zero. Can one determine how many objects there were in the beginning?" The answer is quite elegant and simple, namely:

(a) If sr^i is not divisible by $(r-p)^{i+1}$ for all $i=0,1,2, \ldots, n-1$, the problem has no solution.

(b) If sr^i is divisible by $(r-p)^{i+1}$ for all $i=0,1, \ldots, n-1$, then

$$T_{(o,n)} = \frac{s}{r-p}\left[\frac{\left(\frac{r}{r-p}\right)^n - 1}{\frac{r}{r-p} - 1}\right].$$

2. Lemmas:

It is necessary to establish the following notation in order to clarify later formulas.

Notation: We denote $T_{(j,n)}$ as the remainder after j divisions $(j=0,1,2,3, \ldots, n)$ where n is the total number of divisions to be performed. As immediate consequence, we have the following lemmas:

[1] This research work was partially supported by Contract NsG-381 with the National Aeronautics and Space Administration.

LEMMA 1: $T_{(o,n)}$ = the total number of objects before any divisions.

LEMMA 2: $^{p}/r\ [T(j-1,n)] + \frac{s}{r}$ = the number of objects discarded on the j^{th} division.

LEMMA 3: $T_{(j,n)} = \frac{r-p}{r}\ [T(j-1,n)] - \frac{s}{r}$.

LEMMA 4: $T_{(n,n)} = 0$ since no objects remain after n divisions.

LEMMA 5: $T_{(j-1,n-1)} = T_{(j,n)}$ where $(j=1,2, \ldots, n)$. (1)

PROOF: (of Lemma 5)

(a) Consider the case where j=n. By Lemma 4, $T_{(n,n)} = 0$. Thus, $T_{(n-1,n-1)} = T_{(n,n)}$.

(b) Consider the general case where j=k. Assume $T_{(k-1,n-1)} = T_{(k,n)}$. Show that $T_{(k-2,n-1)} = T_{(k-1,n)}$.

By Lemma 3, $T_{(k-1,n-1)} = \frac{r-p}{r}\ [T(k-2,n-1)] - \frac{s}{r}$ and

$$T_{(k,n)} = \frac{r-p}{r}\ [T(k-1,n)] - \frac{s}{r} \quad .$$

Therefore, $\frac{r-p}{r}\ [T(k-2,n-1)] - \frac{s}{r} = \frac{r-p}{r}\ [T(k-1,n)] - \frac{s}{r}$.

Thus we have $T_{(k-2,n-1)} = T_{(k-1,n)}$ (by induction the lemma is proved).

3. Proof of the Theorem:

In deriving the formula for determing $T_{(o,n)}$ $(n=1,2,3, \ldots)$, we consider the following cases:

Case I. First we consider the case where no objects remain after one division.

From Lemma 4,

$T_{(1,1)} = 0$ and from Lemma 3,

$$T_{(1,1)} = \frac{r-p}{r}\ T_{(o,1)} - \frac{s}{r} \quad .$$

Thus $$\frac{r-p}{r} T_{(o,1)} - \frac{s}{r} = 0$$ and

$$T_{(o,1)} = \frac{s}{r-p} \quad . \tag{2}$$

From Case I we establish the necessity of sr^i being divisible by $(r-p)^{i+1}$ when i=o. Since s must always be divisible by (r-p), we shall denote s by (r-p)s' i.e. s=(r-p)s' where s is a positive integer. Thus (2) becomes

$$T_{(o,1)} = s'. \tag{2.1}$$

Case II: No objects remain after two divisions. From (1) and (2.1) we have that $T_{(1,2)} = T_{(o,1)}$ and $T_{(1,2)} = s'$.

Using lemma 3, $T_{(1,2)} = \frac{r-p}{r} \; [T(o,2) - s']$

$$s' = \frac{r-p}{r} \; [T(o,2) - s']$$

$$T_{(o,2)} = \frac{s'r}{r-p} + s' = \frac{sr}{(r-p)^2} + \frac{s}{r-p} \tag{3}$$

This shows that sr^i must be divisible by $(r-p)^{i+1}$ when i=o,1.

Notice that formulas (2.1) and (3) can be rewritten in the following manner:

$$T_{(o,1)} = s' \left[\frac{\left(\frac{r}{r-p}\right)^1 - 1}{\frac{r}{r-p} - 1} \right] \tag{2.2}$$

$$T_{(o,2)} = s' \left[\frac{\left(\frac{r}{r-p}\right)^2 - 1}{\frac{r}{r-p} - 1} \right] \tag{3.1}$$

Case III: No objects remain after n divisions. From (2.2) and (3.1) we may conclude that

$$T_{(o,n)} = s' \left[\frac{\left(\frac{r}{r-p}\right)^n - 1}{\frac{r}{r-p} - 1} \right] \quad . \tag{4}$$

Obviously sr^i must be divisible by $(r-p)^{i+1}$ for <u>all</u> $i=o,1 \ldots, n-1$. We now show that this formula is true for n equal to any positive integer. The proof is by induction. For $n=1,2$, the formula has been established.

Assume the formula is true $n=k$ and show that it is true for $n=k+1$. By (1), $T_{(o,k)} = T_{(1,k+1)}$ and from the inductive assumption,

$$T_{(o,k)} = s' \left[\frac{\left(\frac{r}{r-p}\right)^k - 1}{\frac{r}{r-p} - 1} \right]$$

where sr^i must be divisible by $(r-p)^{i+1}$, for all $i=o,1, \ldots, k-1$.

Using lemma 3, $T_{(1,k+1)} = \frac{r-p}{r} \; [T(o,k+1) - s']$

Thus

$$s' \left[\frac{\left(\frac{r}{r-p}\right)^k - 1}{{}^{r}/r-p - 1} \right] = \frac{r-p}{r} \; [T(o,k+1) - s']$$

$$T_{(o,k+1)} = \frac{r}{r-p} \left[s' \left[\frac{\left(\frac{r}{r-p}\right)^k - 1}{\frac{r}{r-p} - 1} \right] \right] + s'$$

$$T_{(o,k+1)} = s' \left[\frac{\left(\frac{r}{r-p}\right)^{k+1} - \frac{r}{r-p} + \frac{r}{r-p} - 1}{\frac{r}{r-p} - 1} \right]$$

$$T_{(o,k+1)} = s' \left[\frac{\left(\frac{r}{r-p}\right)^{k+1} - 1}{\frac{r}{r-p} - 1} \right]$$

$$= \frac{s}{r-p} \left[\left(\frac{r}{r-p}\right)^k + \left(\frac{r}{r-p}\right)^{k-1} + \ldots + \frac{r}{r-p} + 1 \right]$$

$$= \frac{s\,r^k}{(r-p)^{k+1}} + \frac{s\,r^{k-1}}{(r-p)^k} + \ldots + \frac{sr}{(r-p)^2} + \frac{s}{r-p} \quad .$$

From the inductive assumption we know that $\frac{sr^i}{(r-p)^{i+1}}$, where $i=o,1,2, \ldots$ $k-1$, are all positive integers. Consequently, $\frac{sr^k}{(r-p)^{k+1}}$ must be a positive integer also. Complete the proof by the induction principle.

Corollary: Under the same assumption of the theorem, if either one of the following statements is true:

(1) s is divisible by $(r-p)^n$

(2) s and r are both divisible by r-p

(3) $p=r-1$

Then, $T_{(o,n)}$ always exists.

To illustrate the application of the formula in several different cases, we consider the following examples:

Example 1: ($n = 2$, $r = 4$, $s = 18$, $p = 1$, $r - p = 3$). In this example, s is divisible by $(r-p)^n$.

$$T_{(o,2)} = \frac{18}{3}\left[\frac{\left(\frac{4}{3}\right)^2 - 1}{\frac{4}{3} - 1}\right]$$

$$T_{(o,2)} = 6\left[\frac{4}{3} + 1\right]$$

$$T_{(o,2)} = 6\left[\frac{4}{3} + \frac{3}{3}\right]$$

$$T_{(o,2)} = 6 \times \frac{7}{3}$$

$$T_{(o,2)} = 14 .$$

Example 2: Both s and r are divisible by (r-p). $n = 3$, $r = 12$, $s = 4$, $p = 10$, $r - p = 2$.

$$T_{(o,3)} = \frac{4}{2}\left[\frac{\left(\frac{12}{2}\right)^3 - 1}{\frac{12}{2} - 1}\right]$$

$$T_{(o,3)} = 2\left[\left(\frac{12}{2}\right)^2 + \frac{12}{2} + 1\right]$$

$$T_{(o,3)} = 2\left[36 + 6 + 1\right]$$

$$T_{(o,3)} = 2 \times 43$$

$$T_{(o,3)} = 86$$

<u>Example 3</u>: When $p = r - 1$, the formula simplifies as follows:

$n = 6,\ r = 3,\ s = 10,\ p = 2,\ r - p = 1.$

$$T_{(o,6)} = 10\left[\frac{\left(\frac{3}{1}\right)^6 - 1}{\frac{3}{1} - 1}\right]$$

$$T_{(o,6)} = 10\left[\left(\frac{3}{1}\right)^5 + \left(\frac{3}{1}\right)^4 + \left(\frac{3}{1}\right)^3 + \left(\frac{3}{1}\right)^2 + \left(\frac{3}{1}\right)^1 + 1\right]$$

$$T_{(o,6)} = 10\left[243 + 81 + 27 + 9 + 3 + 1\right]$$

$$T_{(o,6)} = 10\left[364\right]$$

$$T_{(o,6)} = 3{,}640$$

Finally we consider two examples which have no solution:

<u>Example 4</u>: In some cases sr^i is divisible by $(r-p)^{i+1}$ for <u>some</u> values of $i=o,1,2,\ \ldots,\ n-1$ but not for <u>all</u> values of $i=o,1,\ \ldots,\ n-1$.

(a) $i = o,\ n = 1,\ r = 3,\ s = 2,\ p = 1,\ r - p = 2.$

$$T_{(o,1)} = 1\left[\frac{\frac{3}{2} - 1}{{}^3/2 - 1}\right]$$

$T_{(o,1)} = 1$. The formula holds.

(b) But if $i = 1$, $n = 2$, $r = 3$, $s = 2$, $p = 1$, $r - p = 2$, we have

$$T_{(o,2)} = 1\left[\frac{\left(\frac{3}{2}\right)^2 - 1}{\frac{3}{2} - 1}\right]$$

$$T_{(o,2)} = \frac{3}{2} + \frac{2}{2}$$

$$T_{(o,2)} = \frac{5}{2} \quad .$$

Example 5: If sr^i is not divisible by $(r-p)^{i+1}$ for any value of $i=o,1,2, \ldots, n-1$, the formula fails at the first division. $i = o$, $n = 1$, $r = 4$, $s = 1$, $p = 2$, $r - p = 2$.

$$T_{(o,1)} = \frac{1}{2}\left[\frac{\frac{4}{2} - 1}{\frac{4}{2} - 1}\right]$$

$$T_{(o,1)} = \frac{1}{2} \quad .$$

APPENDIX E

STUDIES ON THE INFLUENCE OF OUTER STREAM VORTICITY ON STEADY AND UNSTEADY LAMINAR FLOWS OF THE BOUNDARY LAYER TYPE

By A. A. Hayday and R. McGraw

In the past decade considerable attention has been directed to some evaluation of the influence of outer stream vorticity on flow regimes which may be treated within the framework of boundary layer theory. The earlier studies on the subject were, with one or two exceptions, approximate, attempting to answer, among others, the following questions of practical interest: 1) What is the influence of outer stream vorticity on dissipative regions, 2) What are the effects of outer stream vorticity on the mixing characteristics of two adjacent streams, and 3) What is the possibility of using vorticity as a rate controlling mechanism for the exchange of physical properties between such streams. These questions (the first in particular) were by no means completely answered. Some of the published papers contained fundamental errors and in one case the findings led to a prolonged controversy. This controversy had to do with the existence (or nonexistence) of certain dynamic effects due to stream vorticity.

In 1962 Van Dyke (1) proposed a systematic treatment which yields (formally) asymptotic solutions of the Navier-Stokes equations for large Reynolds numbers. His work, based on the techniques developed by Kaplun (2), Lagerstrom and Cole (3) and others, places the role of vorticity in the proper perspective in relation to other so-called second order effects. Van Dyke finds that the stream vorticity has in general not only a kinematic but also a dynamic effect on the flow field. Specifically, this dynamic effect appears in the form of a pressure change induced upon the boundary layer by the interaction of the displacement thickness with external vorticity. Thus, at least from the point of view of second order boundary layer theory, the controversy mentioned above appears resolved. This question is of practical interest in problems concerned with the influence of shock-induced vorticity on the boundary layer, (4), (5).

The asymptotic methods, though general, are quite complicated. Furthermore, the published results appear to be limited to steady flows.

It is possible, however, to consider some of the vorticity effects directly, that is, from the point of view of a certain class of exact solutions of the Navier-Stokes equations. In such cases the procedures given in (1) need not be followed. The aim of this program is to obtain a number of such exact solutions for both steady and unsteady flows with particular emphasis on the latter. The examples describe rotational flows near a stagnation point. Solutions of this type are important for several reasons. First of all, they serve as convenient checks of results deduced from asymptotic solutions should the methods for obtaining the latter be generalized to unsteady flows. (Generalizations of this sort shall be studied in the future.) Secondly, the examples studied have a number of features of practical interest, and finally the solutions are of intrinsic interest because they are exact.

Both two-dimensional and three-dimensional flows are considered. Some known solutions for steady flows are extended to include the effects of suction and blowing. (These are of interest in problems connected with boundary layer control and ablation phenomena.) The solutions for unsteady flows cover the following situations: i) a rotational outer stream with fluctuating or decaying vorticity, boundary held fixed, ii) a steady rotational outer stream, boundary oscillating in its own plane, iii) combinations of i) and ii). Some consideration is also given to the effects due to a transverse magnetic field under the usual simplifying assumptions.

To illustrate the nature of the solutions, we consider the simplest examples for two-dimensional flows.

The x,y components of the velocity field for the outer flow are

$$u_1 = a(x - k_1 \frac{\omega_1^\circ \delta_1}{a} e^{-k_2 at}) - \omega_1^\circ (y - \delta_1) e^{-k_2 at}, \quad v_1 = -a(y - \delta_1) \qquad (1)$$

where k_1, k_2 are parameters whose values are zero or one, $\omega_1 \equiv |\mathrm{curl}\ \bar{v}_1| = \mathrm{const}$, a is a constant and δ_1 is the displacement thickness determined in the course of the solution. The superscript zero refers to $|\mathrm{curl}\ \bar{v}_1|$ at $t = 0$. When the flow is steady it is understood that $\omega_1 \equiv \omega_1^\circ = \mathrm{const}$. The above flows are generalizations of the well known potential flow against a half plane athwart the stream to which they reduce (up to the displacement effect) when $\omega_1 \equiv 0$.

Solutions, discussed below, are given for

a) $k_2 = 0, k_1 = 0$ b) $k_2 = 0, k_1 = 1$ c) $k_2 = 1, k_1 = 0$.

Case a) deals with the kinematic effect due to vorticity. Case b) includes the dynamic effect. Case c) deals with flows in which the stream vorticity decays exponentially with time having an initial (maximum)value ω_1°.

The pertinent Navier-Stokes equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\} \qquad (3)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\} \qquad (4)$$

subject to the boundary conditions

$$u(x,0) = 0, \qquad v(x,0) = v_w = \text{const}, \qquad p(x_o,0) = p_o$$

$$\lim_{y \to \infty} u(x,y) = u_1 , \qquad \lim_{y \to \infty} v(x,y) = v_1 \tag{5}$$

where u_1 , v_1 are given by (1).

Admissible solutions are of the form

$$u = ax\, f'(\eta) - \left(\frac{\nu}{a}\right)^{\frac{1}{2}} \omega_1^{\circ}\, h(\eta)\, e^{-k_2 at} , \quad v = -(a\nu)^{\frac{1}{2}} f(\eta) \tag{6}$$

where $\eta = \left(\frac{a}{\nu}\right)^{\frac{1}{2}} y$. Substituting (6) into (2) - (5) yields the equivalent set

$$f''' + ff'' - f'^2 + 1 = 0 \tag{7}$$

$$h'' + fh' - h(f' - k_2) = k_2(\eta - \gamma) - k_1\gamma + k_1k_2\gamma \tag{8}$$

$$f'(0) = h(0) = 0, \qquad f(0) = f_w$$

$$\lim_{\eta \to \infty} f'(\eta) = 1 , \qquad \lim_{\eta \to \infty} h'(\eta) = 1 \tag{9}$$

(We omit the discussion for the pressure variation in the y-direction obtained in the usual manner from (4).)

Some of the numerical solutions to the f-equation are known. These have been extended wherever necessary. The h-equations are linear. The general solutions are the following:

case a)

$$h = A_1 f'' + A_2 f'' \int_o^\eta \frac{G}{f''^2}\, d\eta' \ , \quad G \equiv e^{-\int f d\eta} \tag{10}$$

case b)

$$h = A_1 f'' + A_2 f'' \int_o^\eta \frac{G}{f''^2}\, d\eta' + \gamma f' \tag{11}$$

case c)

$$h = (f' + 1) \int_o^\eta \frac{G}{(f' + 1)^2}\, d\eta' \int_o^{\eta'} \frac{(\eta - \gamma)(f' + 1)}{G}\, d\eta'' + B_1 (f' + 1) \int_o^\eta \frac{G}{(f' + 1)^2}\, d\eta$$

$$+ B_2 (f' + 1) \tag{12}$$

The evaluation of the constants requires some care. ($\int_o^\infty \frac{G}{f''^2}\, d\eta'$ does not exist.) To illustrate the procedure we consider cases a and b. First we rewrite the f-equation in the form

$$f = f'' \int_o^\eta \frac{G}{f''^2} \left[f^{IV}(0) + \int_o^\eta \frac{f''^2}{G}\, d\eta'' \right] d\eta' + \frac{f(0)}{f''(0)}\, f'' \tag{13}$$

(The statement (13) is a consequence of (7) and the boundary condition $f(0) \neq 0$. It is obtained by twice differentiating (7) and properly rearranging and integrating the resulting equation.) The condition $h(0) = 0$ implies that $A_1 = 0$. To evaluate A_2 we compare the derivatives of (13) and (11). At large η we have respectively

$$f' \rightarrow (f^{IV}(0) + \int_o^\infty \frac{f''^2}{G} d\eta)\ (f''' \int_o^\eta \frac{G}{f''^2} + \frac{G}{f''}) \tag{14}$$

and

$$h' \rightarrow A_2\ (f''' \int_o^\eta \frac{G}{f''^2} d\eta' + \frac{G}{f''}) \tag{15}$$

Since $\lim_{\eta \to \infty} f' = \lim_{\eta \to \infty} h' = 1$, we read off

$$A_2 = f^{IV}(0) + \int_o^\infty \frac{f''^2}{G} d\eta = f(0)\,(1 + f(0)\, f''(0)) + \int_o^\infty \frac{f''^2}{G} d\eta$$

The stress at the wall, τ_w, is composed of two parts, τ_{fw} and τ_{hw}. We have

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = \tau_{fw} + \tau_{hw} = \rho(a\ )^{\frac{1}{2}} \times f''(0) - \rho\omega_1 h'(0). \tag{16}$$

Results were also obtained by direct numerical integration of (8). The agreement is excellent for $f(0) \geq 0$. When $f(0)$ is negative, numerical integration is preferable to the procedures implied by (10) - (15).

Other flow examples, are treated basically in the same manner. The solutions are, however, considerably more complicated and shall not be discussed here. The results are now being summarized and submitted in the form of several notes and papers to the appropriate journals.

Appendix E

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APPENDIX F

SIMILAR FLOWS ABOUT AXISYMMETRIC BODIES ROTATING IN A FLUID AT REST

By A. A. Hayday

The concept of similarity has led to a great number of fruitful results in boundary layer theory. The basic idea is contained in the "similarity transformations" - mappings that are one-one except possibly at certain points - which allow us to restate a class of boundary value problems arising from Prandtl's equations in terms of ordinary differential equations. Traditionally, even numerical solutions to such systems of ordinary differential equations are regarded as "exact" and are of great importance in serving as standards for the accuracy of results obtained by various approximate methods. The aim of this study is to enlarge the class of known similar solutions for certain three-dimensional flows of both practical and theoretical interest.

The first phase of this study deals with the following problem: to determine the class of laminar boundary layer flows of the similar type engendered by members of a corresponding class of bodies of revolution rotating uniformly about axes of symmetry in an otherwise undisturbed fluid. This class of bodies is determined (1) and a number of flow examples are studied under the assumptions that the physical properties of the fluid are constant and that dissipation and curvature effects are negligible. We outline briefly the procedures used and some of the more important results.

It is convenient to express the equations for the conservation of mass and linear momentum in a nonrotating coordinate system x,y where x is the distance measured along a meridian curve from the nose of the body and y is the local normal. The x,y velocity components are u,v, the transverse component is w, and r is the body radius. The pertinent equations are

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \qquad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - \frac{w^2}{r} \frac{dr}{dx} = \nu \frac{\partial^2 u}{\partial y^2} , \qquad (2)$$

and

$$u \frac{\partial}{\partial x} (rw) + v \frac{\partial}{\partial y} (rw) = \nu \frac{\partial^2}{\partial y^2} (rw) , \qquad (3)$$

subject to the boundary conditions

$$\begin{aligned} & u(x,0) = 0, \qquad v(x,0) = 0, \qquad w(x,0) = r\Omega \\ & \lim_{y \to \infty} u(x,y) = \lim_{y \to \infty} w(x,y) = 0 \end{aligned} \qquad (4)$$

Introducing Stokes' stream function $\Psi(x,y)$ and imposing a proper similarity criterion leads finally to the equivalent problem,

$$F''' + mFF'' + \frac{2m-1}{3} (G^2 - F'^2) = 0 , \qquad (5)$$

$$G'' + mFG' - \frac{4m-2}{3} F'G = 0 , \qquad (6)$$

$$F'(0) = F(0) = 0, \qquad G(0) = 1, \qquad \lim_{\eta \to \infty} F' = \lim_{\eta \to \infty} G = 0. \qquad (7)$$

The stream function Ψ is related to F by $\Psi(x,y) = L^2(\Omega\nu)^{1/2} (\frac{x}{L})^m F(\eta)$, $w = r\Omega G(\eta)$, the similarity variable being $\eta = y(\frac{\Omega}{\nu})^{1/2} (\frac{x}{L})^{\frac{m-2}{3}}$, and the body radius defining the class of bodies is $r = L(\frac{x}{L})^{\frac{2m-1}{3}}$; L stands for a characteristic length related to $\bar{x}$, the distance measured along the axis of symmetry, by

$$\frac{\bar{x}}{L} = k \cdot \left(\frac{3}{2m-1}\right)^{\frac{6}{4m-8}} \frac{3}{4m-8} \int_0^s \bar{s}^{\frac{k(11-4m)+(k-1)(4m-8)}{4m-8}} (1-\bar{s}^k)^{\frac{1}{2}} \, ds$$

$$\equiv Q(x, k, m) \qquad (8)$$

where the variable s is subject to

$$0 \leq s^k < \left(\frac{2m-1}{3}\right)^2 \left(\frac{x}{L}\right)^{\frac{4m-8}{3}} \leq 1., \quad m \geq 2.$$

We have immediately $L = \bar{x}_{max} [Q(1, k, m)]^{-1}$; $\bar{x}_{max}$ is considered as given a priori. The bodies, of finite length, are cone-like in shape.

The one parameter problem (5) - (7) has several features of intrinsic interest. When m = 2 it covers the important case of the flow due to a rotating disk first studied by Th. von Kármán (2) and later by Cochran (3) and others. (This solution satisfies the full Navier-Stokes equations.) Relabeling the variables we obtain also the boundary layer flow due to a rotating cone first studied by C. S. Wu (4). In both cases the boundary layer thickness is constant. Because of the boundary layer hypothesis, the cone flow analysis breaks down near the tip. For the flows considered herein, this is so in general, but the failure near the tip as well as the over-all character of the boundary layers corresponding to $m > 2$ are altogether different from the boundary layers on spinning cones and disks. When $m > 2$, the boundary layers are shown to be continuously thinning with increasing values of x, the coordinate parallel to the body surface. Near the tip there apparently always exists a region of relatively small velocities not covered by the present equations and this region takes on the character of a continuously thinning boundary layer for sufficiently large x. A properly posed "thick" boundary layer analysis should yield results which join smoothly with the findings contained in (1). A complete analysis of this sort should provide a sound theoretical basis for what appear to be simple experiments on rather basic aspects of boundary layer theory. It is hoped that such experiments will be conducted in the future.

Phase two of this project deals with the solution of a number of heat transfer problems which arise under the flow conditions described in (1). The problems are divided into two categories depending upon the prescribed temperatures at the surface of the bodies. In the first category we discuss the structure of the thermal layer and local heat transfer for surface temperatures either constant or proportional to $(\frac{x}{L})^n$ where n is a positive integer. In the second category we consider more general problems by superposing certain particular solutions of the appropriate energy equations. The details are somewhat lengthy and need not be given here. The results show that the local heat flux (as well as the local friction coefficients) increase markedly with increasing values of m. Qualitatively, this is as expected, since the boundary layer is thinner for higher values of the body parameter, m. A paper (5) has been accepted for presentation at the 1964 Annual Winter Meeting of the American Society of Mechanical Engineers.

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APPENDIX G

STUDIES ON HETEROGENEOUS CONTINUA

By A. A. Hayday

The simplest definition of a (physical) heterogeneous continuum is that it is a material composed of several different constituents. Such material may be solid, liquid or gas. The following are some specific examples: a mixture of inert or chemically reacting gases, an electrically conducting plasma, liquid helium II, and fluids in which solid particles are suspended. The work reported here is concerned with the general structure of the mathematical models for such media and is a continuation of the program which has been in effect at the Research Institute since 1962.

The first phase of this program, a large portion now considered complete, deals with the following problem: to construct directly within the framework of classical field theories (continuum physics) a general set of integral balance equations governing the behavior of heterogeneous continua and from these to deduce rigorously the corresponding differential equations. In much of the past literature on the subject the generalized balance equations are derived more or less intuitively. This accounts perhaps for the various disagreements among the writers on the subject. The first published rigorous generalization is due to Truesdell (1,2) who postulated the differential balance equations for the constituents and from these deduced the corresponding equations for the heterogeneous continuum. Truesdell's work is in a sense a formalism. A direct axiomatization in integral form is obviously preferable.

In the last reporting period the author succeeded in setting forth two such approaches to the theory of heterogeneous continua. The main results of this work are in agreement with one another and also with Truesdell's (1,2) but the formulations are logically distinct.

The first formulation appears to be an acceptable generalization of the classical mathematical theory of one-component continuum. It is an improved version of the author's earlier work (3) and rests for the most part on a (tensorial) superposition proposition concerned with the interrelationship of the relative motions of the constituents and postulated motion of the heterogeneous continuum. The main advantage of this approach is its simplicity and, particularly that the elegant structure of the one-component theory is retained. A condensed version has been submitted for publication.

The second treatment rests on the definition of a heterogeneous continuum as a superposition of simple continua. Each simple continuum is postulated to possess its own motion represented mathematically by a suitable one-one transformation of the three-dimensional Euclidean space into itself. Kinematical quantities associated with the mixture motion are basically defined as certain averages of the component quantities but we show that, at least for certain regions of the $(\bar{x},t)$ space, there exists an alternate representation of such averages related to a certain one-one transformation. For such regions, it is possible to define material volumes for the heterogeneous continuum and state the pertinent axioms in integral form. The axioms are based on a proposition which allows for very general interactions among the components. No electro-magnetic effects are considered; such extensions are left for the future.

The second phase of this project deals with the study and classification of propagating discontinuity surfaces in multicomponent and multiphase systems. The (unpublished) results are essentially the same as in a paper by Kelly (5) though derived from a different and perhaps a preferable point of view. The extent to which the two works are related is now being assessed.

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APPENDIX H

STUDIES IN HYPERSONIC FLOWS

By R. Hermann and J. Thoenes

Three problem areas have been considered during the reporting period.

I. Shock Properties for Dissociated Air

The shock calculations, originally started by J. Yalamanchili, were thoroughly checked, extended and numerically recomputed. The extension of the work done includes the total pressure ratio and the entropy increase across the shock. A final report has been written and published (Ref. 1).

II. Hypersonic Non-Equilibrium Blunt Body Flow

The calculations of non-equilibrium hypersonic flow with free stream dissociation past a circular cylinder were continued. The available calculations (Ref. 2) were extended and improved. The extension includes the derivation of a third set of differential equations by specializing the general equations governing the flow past the body to the stagnation stream line. The resulting set of equations can be integrated along the stagnation stream line to yield "non-equilibrium stagnation point conditions" which are necessary to start the integration along the body surface. Formerly (Ref. 2), due to lack of other information, the stagnation point condition had been assumed to be those for equilibrium flow. This resulted in strong changes of some of the flow variables (for instance, the degree of dissociation) in the immediate neighborhood of the stagnation point, which seems unrealistic. This fact was pointed out in Ref. 3.

The complete equations are presently being reprogrammed for the Univac 1107-Computer. Numerical results are expected in the near future. The former numerical results (Ref. 2) were obtained by a GE 225-Computer, a much slower machine, which made it difficult to obtain the number of iterations needed to decide on the correct solution.

III. Hypersonic Non-Equilibrium Flow Past Pointed Bodies

Recently, pointed shapes for missiles and re-entry bodies have received attention in research and development. Hence, the general compressible flow and thermodynamic equations have been derived by the authors for a curvilinear orthogonal coordinate system particularly suitable for wedge and cone flow. Integrating the equations and using Dorodnitsyn's integral method, the equations have been transformed from the integral form to ordinary differential equations, suitable for numerical analysis. Exact shock relations, including the effect of free stream temperature and oxygen dissociation have been derived and completed. Work in this area is supported by the U.S. Army under Contract DA-01-009-AMC-166(Z).

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APPENDIX I

GROOVE GUIDE MEASUREMENTS

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6.1

GROOVE GUIDE MEASUREMENTS

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Summary

Results of an experimental study of the "Groove Guide" are presented. The guide consists of two parallel conducting strips, each with a centrally located longitudinal groove facing each other. The field distribution, guide wave length, and the attenuation were measured in specially designed test setups. The test setups and measurement circuits are described and the measurement results discussed.

Introduction

In a preceding paper,[1] a nonconventional wave guide, called "Groove Guide", was described. This guide consists of two parallel conducting strips with centrally located grooves facing each other as indicated in Figure 1. The field distribution in the guide, which is a maximum in the center, decreases exponentially toward the upper and lower openings. The guide can thus be open and only a negligible fraction of the energy is radiated.

If waves of the transverse-electric (TE_{01}) fundamental mode with the E-vector parallel to the walls are excited, the guide has reduced attenuation in comparison with the standard rectangular wave guide. Methods for the theoretical consideration of this guide were discussed by Tischer [2,3] based on conformal mapping and by Ruddy [4] by evaluation of the impedance-matching concept.

In the present paper, the results are presented of an experimental study of groove-guide structures carried out at the University of Alabama Research Institute in Huntsville, Alabama. The experimental transverse decrease of the field intensities in direction from the center of the guide, the guide wave lengths, and the attenuation were measured to verify the guide concept and some of the theoretically obtained results. The measurement setup also permits the study of the launching of waves in the groove guide and the effects of discontinuities.

Measurement of Field Distributions

Field-distribution measurements were carried out according to several principles and the results compared. In one case, the field distribution was measured in a specially designed infinite-phase parallel-wall section shown in the upper portion of Figure 2. This figure shows the test setup and the measurement circuitry. The test section has grooves facing each other in the center of the parallel walls. Wooden wedges at the rim between the parallel walls absorb the waves travelling toward the rim and simulate infinite-wall conditions. The waves are launched by a horn radiator into the center part between the parallel walls in the direction of the grooves.

Small holes in the upper wall of the test section permit insertion of a capacitive probe for the determination of the field strength component perpendicular to the walls which decreases exponentially in the direction from the center.

The measurement circuitry consists primarily of a microwave bridge circuit for amplitude and phase comparison with a directional coupler at the transmitting end and an E-H coupler at the output. One of the bridge arms is formed by the grooved-wall test section with probe. Energy is fed by the main guide of the directional coupler into the test section and a fraction of it extracted by the probe. The probe is connected by cable to the comparison arm of the E-H coupler. The other arm of the bridge contains an attenuator and a phase shifter. This arm connects the secondary guide of the directional coupler to the second comparison arm of the E-H coupler. The E-plane arm of the latter contains the matched crystal holder as a zero indicator.

Theoretically, the exponential decrease of the field intensities in the groove guide in transverse direction is given by

$$\alpha_y = [\ (2\pi/\lambda_g)^2 - (2\pi/\lambda_{go})^2\ ] \quad . \qquad (1)$$

In this equation, α_y is the exponential decay factor in Nepers per cm where the values of the guide wave lengths are measured in cm. The guide wave lengths for wave propagation between parallel walls without grooves and the actual groove-guide wave length are denoted by λ_{go} and λ_g respectively. It should be noted that the phase of the field intensity is independent of the distance from the center.

The results of the field strength measurements according to the above principle are shown in the diagrams of Figures 3 to 5. Figure 3 presents contours of constant relative magnitudes of the field intensity in the longitudinal and transverse direction. The amplitude measurements are supplemented by phase measurements. The results of these measurements indicate that the effects of the horn radiator cannot be neglected. Figure 4 shows the results of the phase measurements. The sinusoidal variations of the amplitudes in the longitudinal direction in the contour plots indicate the same effect. Near the horn, the surfaces of constant phases are near-cylindrical. This indicates that radial wave components are super-imposed to the actual groove-guide waves.

The exponential decrease of the field intensities in transverse direction is also shown in the diagrams of Figures 5a and 5b. The diagrams show the magnitudes of the field intensities measured at two frequencies (8.257 GHz and 10.003 GHz) at different longitudinal positions as a function of the distance from the center. The phase measurements also permit determination of the guide wavelength as a function of frequency. With these values the relative exponential decrease of the field intensities in transverse direction can be computed according to Eq. (1).

A second series of field distribution measurements was carried out in a test setup for the determination of the attenuation and the Q-value described later. This test setup consists of a section of an open groove guide with the dimensions H= 16.0, Δh = 0.762, p = 2.286 and Δp = 0.4013 cm placed between parallel conducting walls and forms a groove-guide resonator. A dielectric sphere suspended by a nylon thread can be moved parallel to the walls of the groove guide in the symmetry plane of the guide. The deviation of the resonant frequency Δf of the groove-guide resonator caused by the dielectric sphere as it is moved in the symmetry plane of the guide from the upper opening through the center toward the lower opening is then proportional to the local electric energy,

$$\Delta f = k \left| E \right|^2$$

where E is the local electric field intensity. The ratio of the field intensities E_1 and E_2 at two locations Δy apart in transverse direction is hence

$$\left(\left| E_2 \right| / \left| E_1 \right| \right)^2 = \exp\left(-2\alpha_y \Delta y\right) = \Delta f_2 / \Delta f_1 , \quad (2)$$

since $E_2 = E_1 \exp(-\alpha_y \Delta y)$. The quantity α_y is the decay constant of the field intensity in transverse direction.

The results of these measurements are plotted in Figures 6 and 7 on semi-logarithmical paper for the frequencies 8.257 GHz and 10.003 GHz for a dielectric sphere and a dielectric disc as probing elements. The curves show the exponential decrease which is characterized by a straight line in the region excluding the center of the guide near the groove and the upper and lower near-edge regimes.

Figure 8 shows the results of an evaluation of the diagrams in Figures 6 and 7. It shows the transverse decay factor α_y obtained from the slopes of the curves plotted in these diagrams. The values are indicated by the circles. As a comparison, the values obtained from the measurement of the guide wave length according to Eq. (1) are plotted in curve A.

Guide Wave Length

The guide wave length λ_g of the groove guide is somewhat smaller than that of a parallel-wall guide without grooves λ_{go}. This reduction is primarily caused by the storage of magnetic energy in the groove related to the longitudinal magnetic field component.

As an approximation, the guide wave length can be computed according to a method outlined in Reference 1. The method can be refined by considering an equivalent H-guide containing a slab of material with an anisotropic permeability such that the longitudinal component of the relative permeability tensor only deviates from the value 1. The various values of the guide wave length are plotted in Figure 9 as a function of frequency. The measured values of the guide wavelength of the groove guide are shown in C and compared with those obtained by the above approximate computation method. It should be noted that the approximation is good at lower frequencies near cutoff.

Attenuation Measurements

The attenuation of the groove guide was determined by a resonance method. Hereby, the attenuation is being computed from the measured Q-value of a resonator which contains the wave guide under consideration. The groove-guide resonator contains a section of the guide placed between two parallel conducting walls. The walls reflect the waves traveling in and outside along the groove guide at the input and output ends of the guide section. Energy is fed into the resonator by a small coupling hole in the center of the end wall of the guide. The coupling hole connects the resonator to the end of a rectangular X-band wave guide and to a klystron as energy source. Figure 10 shows a photograph of the resonator and the measurement circuitry.

Several methods for the measurement of the Q-value were tested such as reflection measurements at the input and probe measurements with swept frequency. The probe measurements gave the most reliable results. The probe consisted of a small capacitive antenna placed near the end of the groove-guide section and intruding slightly into the region between the parallel walls. The frequency was swept by a sawtooth signal applied to the reflector of the klystron. Low coupling was used to eliminate loading of the resonator by the wave guide feeding energy into the resonator and by the probe. Displaying the field intensity on an oscilloscope with the saw tooth as horizontal deflection voltage, the Q-value can be obtained by the evaluation of the resonance curve. The Q-value is given by $Q = fo/\Delta f$ where fo is the resonant frequency and Δf is the frequency difference between half-power points.

The attenuation of the guide is related to the Q-value of the guide by

$$\alpha \text{ [Nepers/cm] } = \frac{1}{2} \frac{\beta}{Q} \left(\frac{\lambda_g}{\lambda_o}\right)^2 , \qquad (3)$$

where $\beta = 2\pi/\lambda_o$ and λ_o is the operational frequency.

The test setup was also used for the determination of the guide wave length by finding the number of half-wave lengths along the resonant groove-guide section at consecutive resonant frequencies in the X-band region. Field strength measurements were also carried out in the resonator. The relative field strength within the guide was determined by moving a dielectric sphere through the guide and by measuring the deviation of the resonant frequency as a function of the position of the sphere as discussed previously.

The results of the Q-value measurements are presented in Figure 11. It shows the Q-value of the groove guide (A) as a function of frequency. For comparison, the Q-value of a resonator containing a section of a rectangular guide of equal length and machined from the same material is plotted below (B).

The attenuation of the groove guide (A) computed from the Q-value according to Eq. (3), is presented in Figure 12 in comparison to that of the rectangular guide plotted in the same figure (B).

The results agree with theory which shows that the groove guide has approximately the same attenuation as a parallel wall guide of the same width. It should be noted that the advantages of the groove guide are in the millimeter-wave region where the width of the guide can easily be increased to oversize dimensions in comparison to the standard rectangular guide. In this frequency range the improvement of the attenuation becomes more pronounced.

Conclusions

The experimental study of the groove guide with regard to field distribution, guide wave length, and attenuation shows results which are in agreement with the predicted properties and with the theoretical data. The attenuation of the guide is of particular interest. It is found to be approximately equal to that of a parallel wall guide of equal width at the same frequency. For obtaining full benefit of the reduction of the attenuation, the guide has to be oversized, which can be considered as an advantage at millimeter waves, and the difference between the two guide wave lengths λ_{go} and λ_g should be as large as possible.

The study indicates also that the effects of the launching of the groove-guide waves and the excitation of other wave modes needs further study. Optimization of the structure to give minimum attenuation is another topic for additional study. Most of these can be carried out in test setups used for the experiments described after some modification.

Acknowledgement

The authors wish to express their appreciation to Mr. Hung Yuet Yee for his assistance in the evaluation of results of the measurements.

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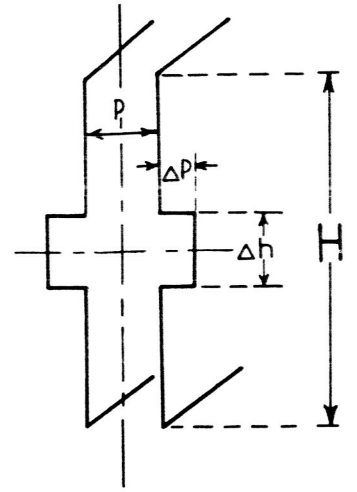

Figure 1 - Groove guide cross section.

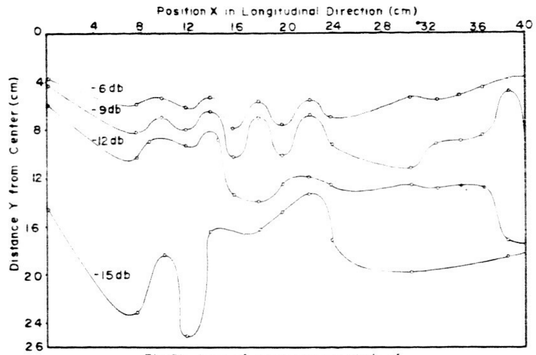

Fig 3b Loci of constant magnitude of field intensity at 10.003 GHz.

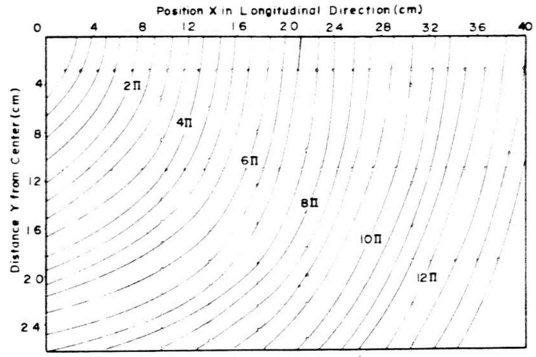

Fig 4a Loci of constant phase of field intensity at 8.257 GHz.

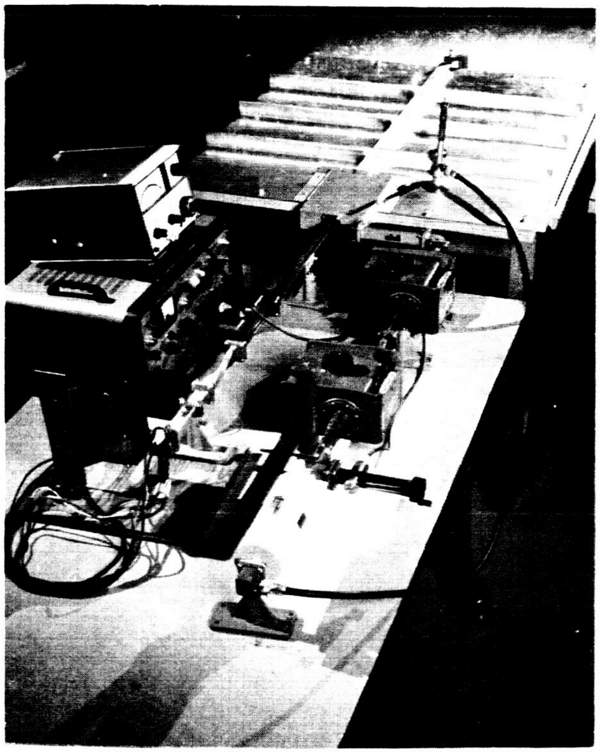

Figure 2 - Test setup for the measurement of the field distribution.

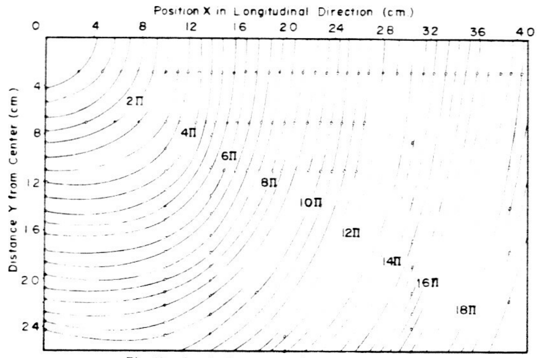

Fig 4b Loci of constant phase of field intensity at 10.003 GHz.

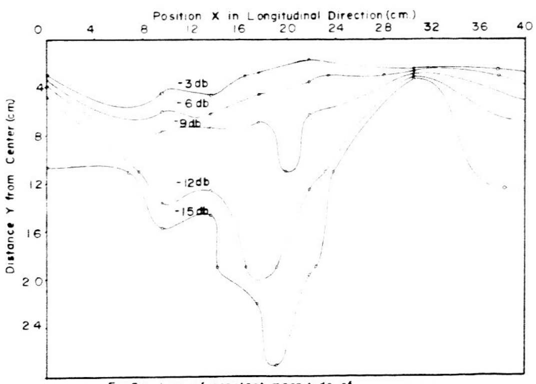

Fig 3a Loci of constant magnitude of field intensity at 8.257 GHz.

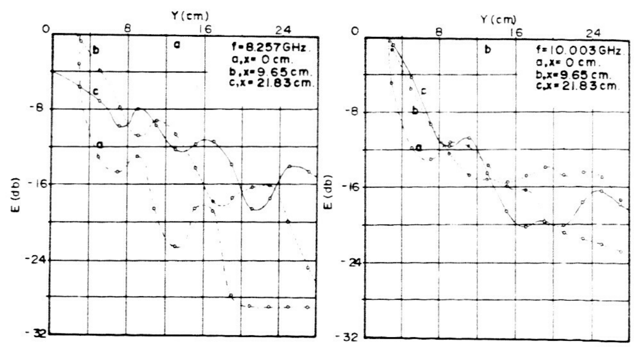

Fig 5 Relative field intensity |E| versus distance from the center axis X.

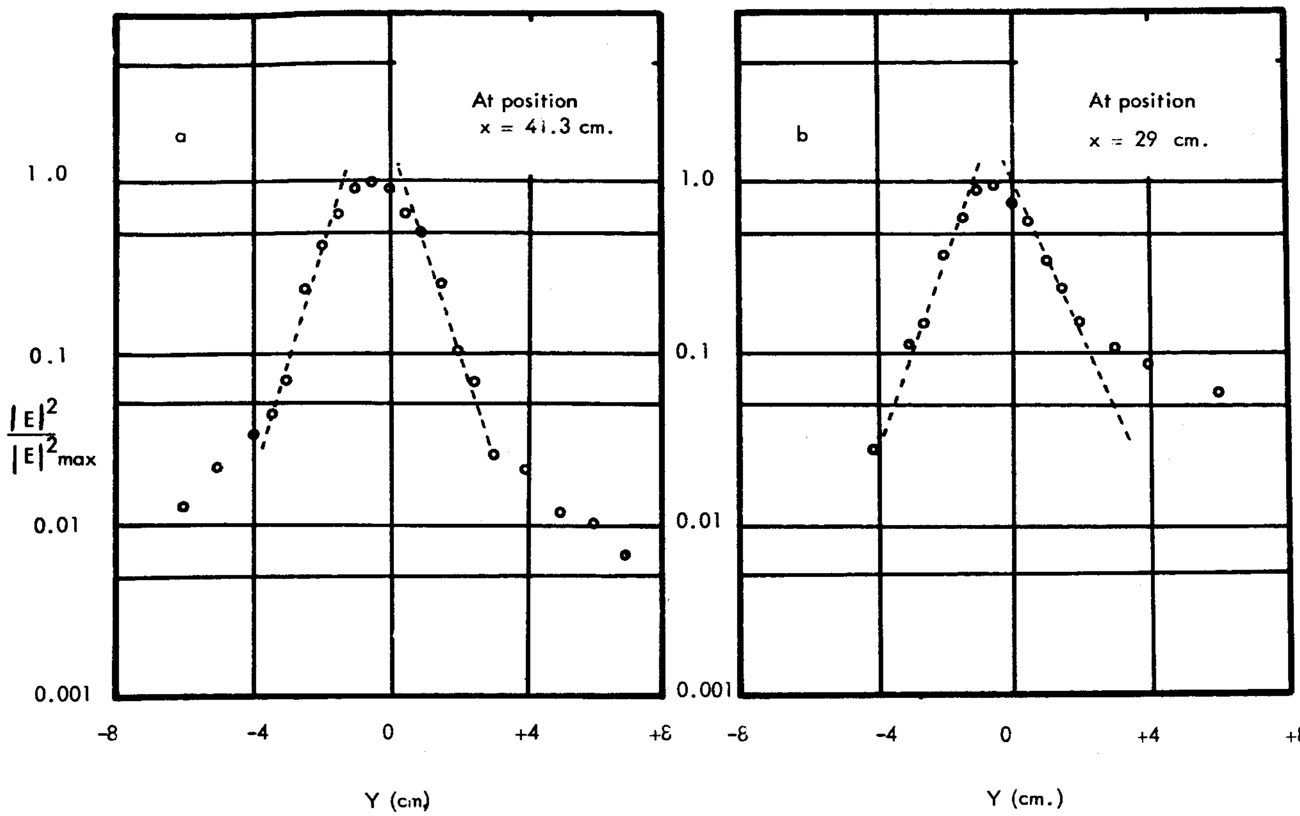

Figure 6 - Relative field intensity as a function of the distance from the center axis of the groove guide for, a (fo = 8.253 GHz) and b (fo = 10.033 GHz) measured with a dielectric disc.

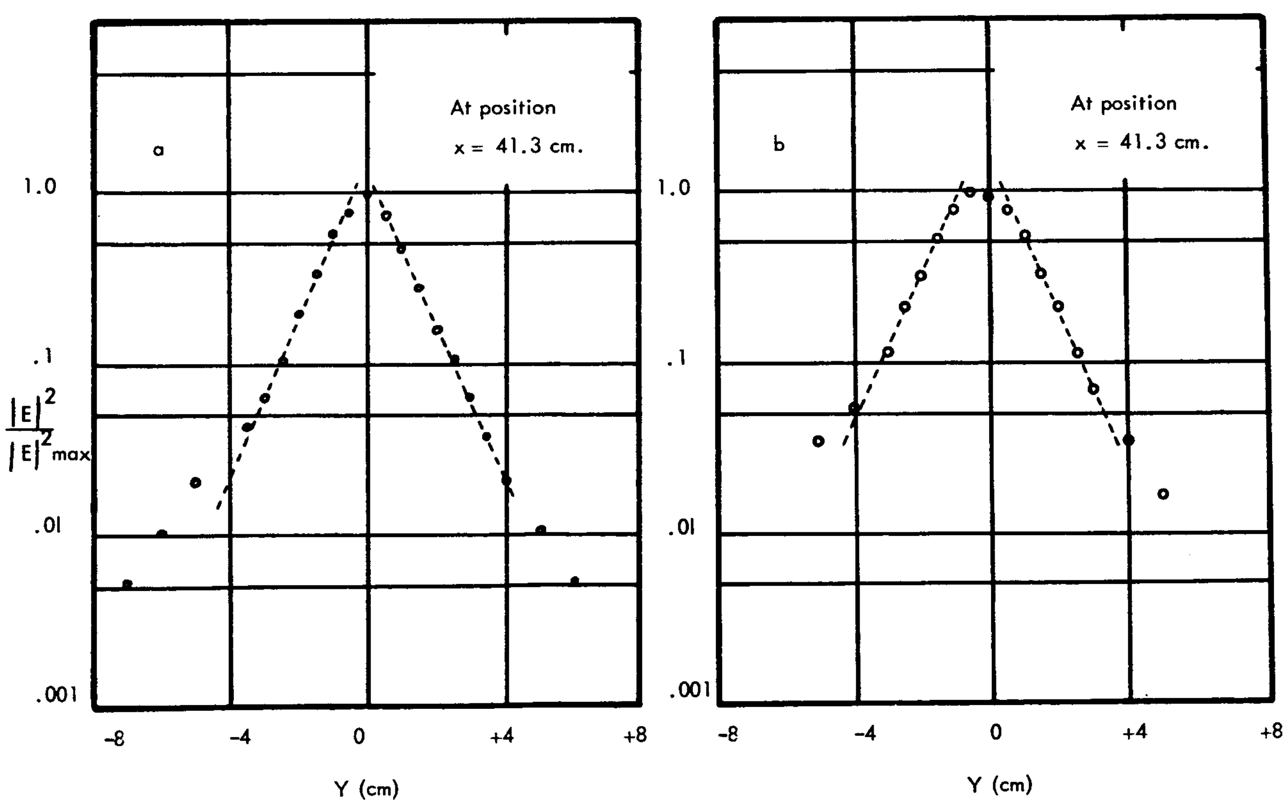

Figure 7 - Relative field intensity as a function of the distance from the center axis of the groove guide for, a (fo = 8.253 GHz) and b (fo = 10.026 GHz) measured with dielectric sphere.

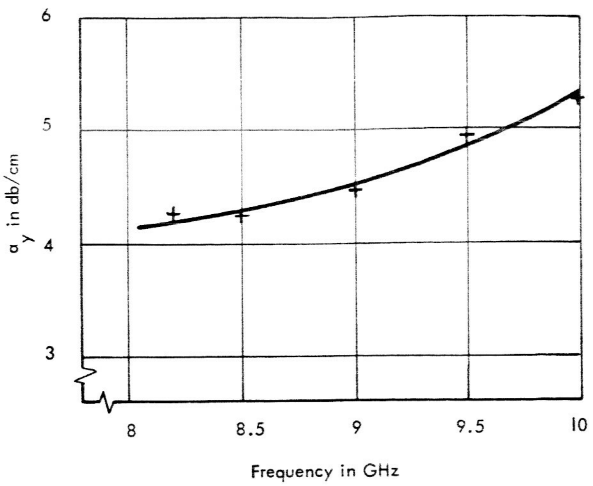

Figure 8 - Transverse decay factor α_y.

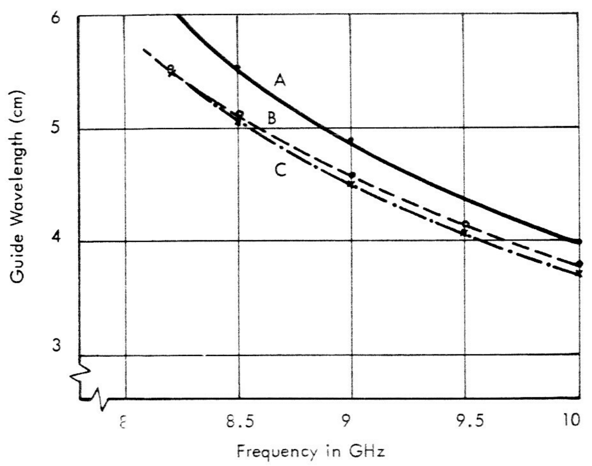

Figure 9 - Guide wave lengths
A Parallel walls without grooves.
B Groove guide, computed
C Groove guide, measured

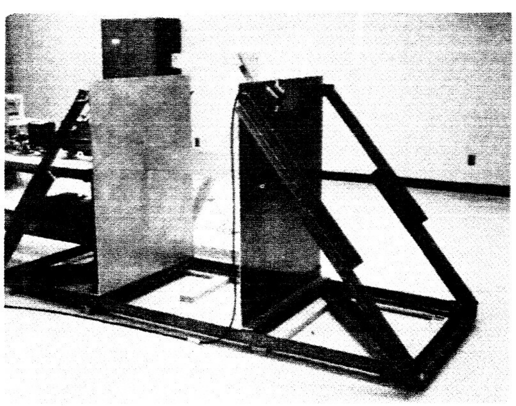

Figure 10 - Groove-guide resonator.

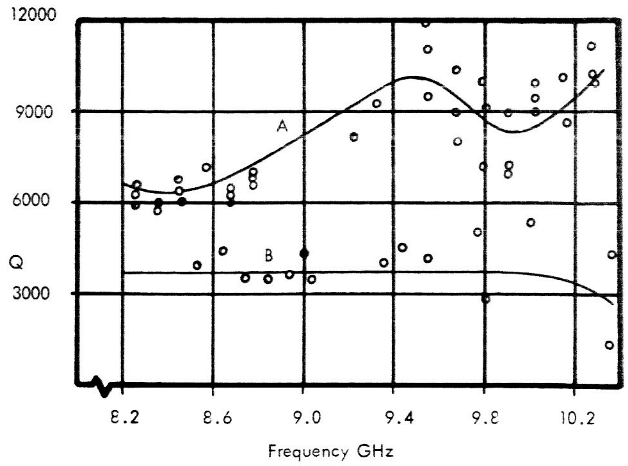

Figure 11 - Q-value versus frequency.
A Groove-guide resonator.
B Equivalent rectangular guide.

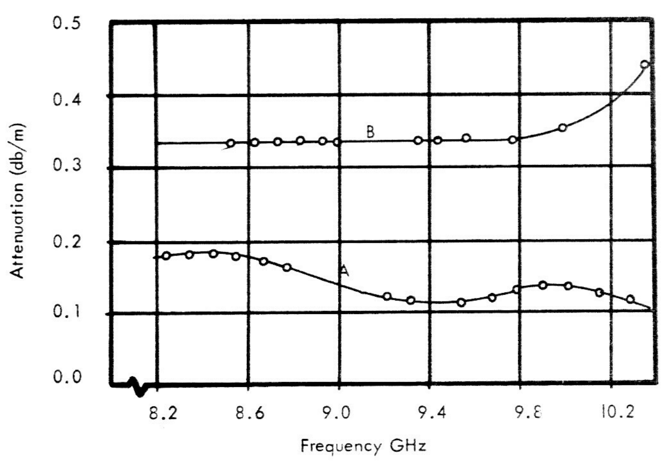

Figure 12 - Attenuation versus frequency.
A Groove guide.
B Rectangular guide.

APPENDIX J

PROJECTS IN MATHEMATICAL THEORY OF AUTOMATIC CONTROL

By C. D. Johnson

During the reporting period, the author continued his research in the mathematical theory of automatic control. The particular areas of investigation and the work accomplished are summarized below.

1. Singular Solutions in Problems of Optimal Control - The subject of "singular solutions" in classical and non-classical variational (optimal control) problems has received considerable attention since the publication of the tutorial paper by Johnson and Gibson [1]. During the reporting period, the author continued work on the preparation of a detailed summary of the current state of the art of this subject including some recent results obtained at the UARI. This summary, which was initially supported under NASA Grant NsG-381, is presently being supported by the Army Missile Command under Contract DA-01-009-AMC-165(z), and will appear as Chapter 6 of the forthcoming text, "Advances in Control Systems: Theory and Application, Vol. II" edited by C. T. Leondes and published by Academic Press.

2. Optimal Bang-Bang Control - During the reporting period, the paper: "Optimal Bang-Bang Control with Quadratic Performance Index" by W. M. Wonham and the author, appeared in the A.S.M.E. Transactions, Journal of Basic Engineering (March 1964). However, most of the research reported in this particular article was performed prior to March 1, 1964.

3. A Problem of Letov in Optimal Control - A. M. Letov is recognized as one of the outstanding Soviet theoreticians in the subject of automatic control theory. In 1960-1962 Letov published a series of papers in which he developed a general solution for a certain class of optimal control

problems [2]. The simplicity of the mathematical structure of Letov's solution was very appealing to both theoretical and practical control engineers and numerous extensions of Letov's results have since appeared in technical control journals in both East and West. In late 1963, the author and Dr. W. M. Wonham of the Research Institute for Advanced Studies at Baltimore, discovered a basic error in Letov's original solution. A detailed account of this error, and a discussion of the correct method of solution, was prepared during the period September 1963 through May 1964 and in June 1964 a paper on this subject was delivered by the author before the Fifth Joint Automatic Control Conference at Stanford, California. The results reported in this paper are currently being revised and extended and will be published in the March 1965 issue of the A.S.M.E. Transactions, Journal of Basic Engineering. The work described above was initially supported by the NASA Grant NsG-381 and, after April 1, 1964, was supported by NASA under Contract NAS8-11231.

4. Transformations to Canonical Form - In the study of n^{th} order linear automatic control systems it is often convenient, for mathematical reasons, to transform the original vector-matrix differential equations to a standard or canonical form via a non-singular linear transformation. One of the more useful canonical forms is the co-called "phase variable" form in which: $dx_i/dt = x_{i+1}$, $i = 1, \ldots, n-1$. Although the existence of a transformation to canonical (phase-variable) form is easily established, it is not always easy to actually write out the transformation explicitly. During the reporting period the author and Dr. Wonham continued their study of this problem and succeeded in developing an explicit

expression for the required transformation in terms of a modal and Vandermonde matrix of the original plant matrix. This work was initially supported by NASA Grant NsG-381 and, after April 1, 1964, was supported by NASA under Contract NAS8-11231. The results of this study appeared in the paper, "A Note on the Transformation to Canonical (Phase-Variable) Form", IEEE Transactions P.T.G. on Automatic Control, July 1964.

5. Optimal Control with Quadratic Performance Index and Fixed Terminal Time - The optimal control of a linear stationary regulator with quadratic performance index and fixed terminal time has been studied by several authors. Previous solutions given for this problem have been obtained by assuming time dependent quadratic solutions for the Hamilton-Jacobi equation. By this means, the optimal control is obtained in the form of a linear feedback controller with time varying gains. This form of solution, although mathematically correct, is physically unrealizable because the time varying gains are required to become infinite as the specified terminal time is approached.

During the reporting period, the author continued his study of an alternate method for solving the Hamilton-Jacobi equation for this class of problems. This method is based on the theory of complete integrals and leads to an optimal feedback controller which is nonlinear and time-invariant. Certain parameters in the nonlinear controller are functions of the initial time and initial state of the plant. This nonlinear optimal controller is mathematically equivalent to the conventional time varying linear controller but does not involve the troublesome infinite gains.

More recently, the results of this study (now supported by NASA under Contract No. NAS8-11231) have been summarized in a paper which will appear in the forthcoming October issue of IEEE Transactions, PTG on Automatic Control.

6. Invariant Hyperplanes for Linear Dynamical Systems - The study of systems of linear differential equations is an important aspect of linear control theory. During the reporting period, the author initiated a study of "invariant sets" for linear differential equations with particular emphasis on invariant hyperplanes for antonomous systems. From this investigation, the relationship between eigenvalues of the plant matrix and invariant hyperplanes has been developed. It appears that this information will be useful in establishing bounds on the "transient" solution of antonomous linear systems and may provide further insight into Kalman's concept of "controllability". These results are currently being extended, through support from NASA Contract No. NAS8-11231, and will appear in a future publication.

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APPENDIX K

PULSE MODULATED SYSTEM WITH ADAPTIVE THRESHOLD

by

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Introduction

The probability of error in the detection of coded pulse-type signals in the presence of noise is reduced when an adaptive threshold level is used instead of a constant threshold level.

In pulse detection without memory the amplitude of the total received signal is compared to a fixed threshold. The statistical properties of the transmission can be improved in an adaptive scheme with memory, where the threshold varies in relation to the noise signal at a time prior to the signal detection.

An RC filter is used to eliminate the high frequency noise at the cost of an acceptable distortion of the pulse. The optimum time constant is larger for adaptive threshold where significant correlation between successive pulses is required for partial noise prediction than for constant threshold where pulse overlap is undesirable.

The adaptive threshold scheme with RC filter is analyzed in the case of white gaussian noise, band limited or not. Since the signal is in the form of pulses, gating can be used to eliminate a large part of the noise.

Detection Schemes Under Consideration

The error probability for constant threshold and adaptive threshold are compared successively for the following cases:

A. white noise of normal distribution, no gating, RC filter

B. white noise of normal distribution, gating, RC filter

C. band limited, white noise of normal distribution, gating, RC filter.

In each case the probability distribution remains normal as evident without gating and as shown with gating.

Let s_1, n_1, s_2, n_2, be respectively the signal and noise at time t and t+T (T delay between two signals), the joint and conditional probability densities can be computed.

The parameters of interest are the power spectrum of the noise, the amplitude of the pulse before filtering, the width of the pulse and the time constant of the filter.

In the minimization of the probability of error, two types of constraint are used, either limited amplitude and variable width or limited energy of the pulse.

Although a continuous variation of the threshold gives the best results, a discrete variation is also investigated because of its simplicity.

Effect of the Filter on the Noise

In the adaptive scheme the noise must be predicted and the threshold varied accordingly. If T is the interval between two signals, the prediction of the noise at time t + T, knowing the noise at time t, is based on the value of the auto-correlation function R (T). Besides filtering the RC network increases R (T) and therefore increases the accuracy of the prediction.

Effect of the Filter on the Pulses

The RC filter changes the shape and the width of the pulses. The amplitude increases in a sequence of overlaping pulses because the initial charge of the condenser is increasing every time. Since the increase from the first to the second impulse is much more than the increase of the following impulses, only two types of impulses will be considered:

(1) No previous impulse,

(2) One or several previous impulses. The assumption here is that the amplitude increases very little between the cases of one or many previous impulses.

Probability distribution

The signal s_2 at time t + T must be compared to a threshold A_T in order to decide if there is a pulse or not. The signal s_1 at time t is assumed known. It is therefore necessary to determine the conditional probability $p\ (s_2/s_1)$.

The cases of no gating with white noise, gating with white noise, no gating with band limited white noise, gating with band limited white noise, are examined successively.

No Gating With White Noise

The probability of error is a function of the choice of the threshold A_T. The cases of constant threshold and of several adaptive threshold are compared.

Let s_o, s_1, s_2, n_o, n_1, n_2 be the signal and noise respectively at time t - T, t and t + T. Assuming additive noise

$$s_1 = n_1 + \gamma_1 A_2 + \gamma_o \rho A_2$$

$$s_2 = n_2 + \gamma_2 A_2 + \gamma_1 \rho A_2$$

where A_2 is the amplitude of a first pulse (after filtering), γ_o, γ_1, γ_2 are 0 or 1 respectively depending on whether or not a pulse was transmitted at time t - T, t or t + T. ρ is the correlation coefficient.

A. Constant threshold, $A_T = A_2/2$, non-negligible correlation

The average probability of error is

$$\bar{E} = \int_{-\infty}^{+\infty} \frac{e^{-v^2/2}}{\sqrt{2\pi}} \int_{u_1}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} \, du \, dv \qquad (5)$$

$$u_1 = \frac{A_1 \; T \; (1 - e^{-ky}) \; (1 - \gamma_1 2 e^{-y})}{\sqrt{\eta} \; \sqrt{y} \; \sqrt{1 - e^{-2y}}} - \frac{e^{-y} v}{\sqrt{1 - e^{-2y}}}$$

where $u \equiv \dfrac{n_2 - e^{-y} n_1}{\sqrt{\dfrac{\eta y}{4T}} \sqrt{1 - e^{-2y}}}$, $v \equiv \dfrac{n_1}{\sqrt{\dfrac{\eta y}{4T}}}$

η is the power density spectrum of the white noise $y = T/RC$ $K = \theta/T$ ratio of the width θ to the period T, for the square pulse before filtering.

In the standard case of constant threshold $A_T = A_{2/2}$ of negligible correlation coefficient ($\rho \approx 0$), formula 5 reduces to the well known formula:

$$\int_{n_2 = A_{2/2}}^{\infty} \frac{1}{\sqrt{2\pi} N} e^{-n_2^2/2N^2} dn_2$$

where N is the r.m.s. value of the noise.

B. Adaptive threshold, optimum choice, $A_T = A_{2/2} + \gamma_1 \rho A_2 + \rho n_1$

The threshold is varied to take into account the sequence of the pulses and the predicted noise.

The average probability of error is

$$\bar{E} = \int_{-\infty}^{+\infty} \frac{e^{-v^2/2}}{\sqrt{2\pi}} \int_{u_1}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} \, du \, dv \qquad (7)$$

$$u_e = \frac{A_2}{2N\sqrt{1-\rho^2}}$$

C. Adaptive threshold, discrete variation

The adaptive threshold (seeB) is optimum except for the assumption that the memory of the system is only one cycle. The threshold is increased by a discrete step, ρA_2, if there was a pulse before and by a variable step, ρn_1, equal to the predicted noise.

For simplicity it may be desirable to approximate ρn_1 by 0, $\triangle$ or $-\triangle$ depending on whether n_1 is small ($-m < n_1 < m$), positive ($n_1 \geq m$) or negative ($n_1 \leq m$).

D. Nonlinear RC network.

If a nonlinear RC network is used where the time constant of the discharge is more than the time constants of the charge, the probability of error decreases.

Minimization of the Probability of Error

It was shown that the average probability of error is a double integral, function of the signal to noise ratio, the time constant RC (or $y = T/RC$), the relative width of the pulse K and the choice of the threshold level.

Given physical constraints, on the maximum amplitude or on the maximum power per pulse, and a signal to noise ratio, the values of y and K which minimize the average probability of error must be obtained. The signal to noise ratio and the relative width of the pulse K, are treated as parameters, while y, i.e., the time constant of the RC network, is treated as a variable. The calculus of variations or a computer can be used:

a. Minimization using calculus

The problem can be solved only for case B. A relation between y and K is obtained, this relation is independent of the signal to noise ratio.

b. Minimization using a digital computer

Using a computer, the probability of error can be computed for the different cases of adaptive threshold. After a change of variables all the integrals are normalized. Different values of S, (S signal to noise ratio), K and y are used. Given S and K the average error probability is read as a function of y, and the minimum can be read on the curve.

White Noise With Gating

It is shown that the power density of the noise η must be replaced by a smaller value $\eta\,(1 - e^{-2\theta/RC})$ in the formula of 5. In other words the effect of gating increases the signal to noise ratio by a factor $\dfrac{1}{1 - e^{-2\theta/RC}}$.

APPENDIX L

Measurement of Orbital Parameters

by

O. R. Ainsworth
C. M. Chambers, Jr.
F. J. Tischer

This research project deals with optimization of the measurement procedures for tracking of space vehicles. Dynamic models and correction procedures are being studied.

For the dynamic model, the best two body problem is solved exactly. The perturbations are formulated as a first order Hamiltonian system and are solved by application of a generalized Newton-Raphson iterative operator in the metric space of the Hamiltonian variables. This procedure eliminates the difficulty of numerically integrating the non-linear perturbation equations, and allows relatively large, constant step length to be used. The best two body problem is rectified when the perturbations exceed specified limits.

The differential correction problem is formulated as a Wiener problem . An optimal linear estimate (filter) of the errors in the dynamic variables is obtained from the observed variables by Kalman's method of orthogonal projection of the dynamic variables onto the space of the observed variables. This procedure allows a correction to be made at each observation and avoids the impractical necessity of waiting until sufficient data is accumulated to overdetermine the least squares system of differential correction equations. Predicted errors are obtained through a state transition matrix and the best time to make a measurement is determined. This research is presently being programmed for UNIVAC 1107 computer.

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[1] R. E. Kalman, Jour. of Basic Engr., 82 , 35 (1960).

APPENDIX M

BIOGRAPHIES OF NEW RESEARCH INSTITUTE PERSONNEL

During this reporting period the academic and full-time research staff of the Research Institute has been greatly strengthened by the addition or prospective addition of the following persons:

Dr. William F. Arendale. Dr. Arendale accepted an appointment effective September 1, 1964, as Assistant Director of the Research Institute and Professor of Chemistry at the University of Alabama in Huntsville. Dr. Arendale, who was awarded his M. S. in chemistry and his Ph. D. in chemical physics at the University of Tennessee, comes to the Research Institute from Thiokol Chemical Corporation, Huntsville, Alabama, where he progressed during thirteen years through the successive positions of Chief Project Chemist, Head of Research Department, Director of Research, Assistant to General Manager, and Technical Director. Dr. Arendale is beginning work in molecular chemical physics with emphasis on the high temperature area.

Dr. Nadeem Fawzi Audeh. Dr. Audeh was appointed on July 27, 1964, as Visiting Associate Professor, Department of Electrical Engineering, and Research Associate, Research Institute . He was awarded both his M. S. and his Ph. D. in Electrical Engineering at Iowa State University, and came to the Research Institute from Los Angeles State College where he was Associate Professor, Engineering Department. Dr. Audeh has begun work in electromagnetics, particularly in the microwave area. During the academic year his time will be divided between teaching and research.

Dr. Chander Perkash Bhalla. Dr. Bhalla accepted an appointment as Assistant Professor of Physics, effective September 14, 1964. He earned his M. S. degree in physics at Punjab University, India, and his Ph. D. in physics at the University of Tennessee. Dr. Bhalla comes to the University from the Westinghouse Electric Corporation, Atomic Power Division, Pittsburgh, where he was Senior Scientist in the Reactor Development Department beginning in 1960. Dr. Bhalla will do part-time research in theoretical nuclear physics, initially in the theory of beta decay.

Dr. Robert L. Causey. Dr. Causey has accepted an appointment as Associate Professor of Mathematics, University of Alabama, to be effective about November 1, 1964. He obtained his B. S. in physics and his M. S. in mathematics at the University of Kentucky, and his Ph. D. in mathematics at Stanford University. For the past five years Dr. Causey has been Research Scientist, Aerospace Sciences Laboratory, Research and Engineering Division, Lockheed Missiles and Space Corporation, Palo Alto, California. Because of his qualifications and experience in the field of numerical analysis, Dr. Causey will start a research program in areas such as celestial mechanics, which require the use of a large scale computer.

Mr. Juang-Chi Chang. Mr. Chang has accepted an appointment effective September 28, 1964, as Research Associate. He obtained his M. S. in Electrical Engineering at the University of British Columbia, Vancouver, Canada, and has completed his Ph. D. course work and other requirements in Electrical Engineering at Iowa State University, where he expects to obtain his Ph. D. degree in December 1964. He will work in electromagnetics with special emphasis on tracking by electronic means.

Mr. Chi-Fan Chen. Mr. Chen has accepted an appointment as a Research Associate to be effective November 1, 1964. Mr. Chen obtained his M. S. degree in Electrical Engineering at the University of Pennsylvania. He comes to the Research Institute from Christian Brothers College, Memphis, Tennessee, where he is Professor of Electrical Engineering. He will work in the mathematical theory of control, particularly in stability theory.

Dr. Juerg-Heinrich Kallweit. Dr. Kallweit joined the Research Institute staff on June 1, 1964 as a Senior Research Associate. He obtained his M. S. degree in physics and his Ph. D. in experimental physics from the Technische Hochschule at Braunschweig, where he was an Assistant Professor. Afterwards, he was a tthe Feldmuehle A.G., (an industrial concern), at Duesseldorf, Germany where he was Branch Chief of research in physics. He will do work in the fields of high-vacuum physics, surface physics, infrared spectroscopy and/or research in high-temperature plasma jets using optical measurement techniques.

Dr. Wilhelm K. Kubitza. Dr. Kubitza has accepted an appointment, to be effective about February 1, 1965, as Head of the Structural Mechanics Laboratory at the Research Institute, and Professor, Department of Engineering Mechanics in the College of Engineering. He obtained his Ph. D. from Washington University in St. Louis, Missouri. Dr. Kubitza is at present Professor, Department of Civil Engineering at Kansas State University, where he has been since 1953. Upon his arrival at the University of Alabama, his time will be divided between the Research Institute, where he will activate the Structural Mechanics Laboratory, and the graduate instructional program.

Mr. Manfred Johannes Loh. Mr. Loh joined the Research Institute on August 24, 1964 as an Aeronautical Research Engineer. He obtained his M. S. degree in Aeronautical Engineering (Diplom Ingenieur) at Technische Hochschule Aachen, West Germany. Mr. Loh came to the Research Institute from the Messerschmitt Aircraft Corporation in West Germany where he was a Branch Chief for the past three and one-half years. He will work in the areas of supersonic aerodynamics, hypersonics and re-entry.

Mr. Ferdinand H. Mitchell. Mr. Mitchell had held the position of Research Assistant at the Research Institute since June, 1963 while working toward his Ph. D. degree in physics. He had previously obtained his M. S. in physics from the University of California at Los Angeles. Mr. Mitchell completed all his requirements for the Ph. D. in September, 1964 and is being appointed Research Associate, effective October 1, 1964. He will continue to work in the area of electromagnetic wave propagation.